ELEMENTARY

MENSURATION AND SURVEYING

(Including Plane Practical Geometry)

BY

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PREFACE.

The book has been written with a view to supply the want of a treatise with an elementary treatment of the allied subjects of Practical Geometry, Mensuration and Chain Surveying.

The author acknowledges the help he has received from Mr. S. Deb Gupta of the Pabna Technical School. He also acknowledges that the Mensuration exercises have been chosen from various Examination Papers, set in India, admirably collected in Pierpoint's Mensuration.

<div align="right">A. N. SEN</div>

SOME BOOKS

By

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M. A. S. M. E. (U. S. A.), A. M. I. E. (Ind.)

1. **School Geometry.**
2. **Elementary Mechanics.**
3. **Plane Practical Geometry.**
4. **Elementary Mensuration.**
5. **Prathamic Patiganit.**

———.o.———

ELEMENTARY
MENSURATION AND SURVEYING

(Including Plane Practical Geometry)

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SECTION 1. •

INTRODUCTORY.

Definitions and Propositions of Geometry. Tables of length, area and volume. Formulae.

1. Mensuration deals with the methods of estimating lengths, areas and volumes, by the practical use and application of the rules and formulae derived from Geometry, Trigonometry and other branches of mathematics. The proofs of these rules and formulae are, however, not considered to be within the scope of the subject as treated here, and only the simplest cases have been discussed. The definitions of quantities and the more important propositions of Geometry, tables of units of length, area and volume and formulae of mensuration, of frequent use, are capitulated below for ready reference.

2.   Definitions of Geometry. A *point* is that which has position but no magnitude.

A *line* (straight or curved) has length but no breadth or thickness. A *right* or *straight line* has the same direction throughout its length and is the shortest distance between two points at its extremities. A curved line changes its direction between points within its extremities.

A *surface* has length and breadth but no thickness. Its boundaries are lines. It may be flat or curved. The extent of a surface, is called its *area*. A *plane surface* is perfectly flat and even ; if any two points are taken on the surface, the straight line between them lies wholly in that surface.

A *solid body* has length, breadth, and thickness.

Solids, surfaces, lines, and points are related to one another as follows :—

1.   A solid is bounded by surfaces.

2.   A surface is bounded by lines. Surfaces meet along lines.

3.   A line is terminated by points. Line meet in points.

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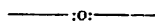
When two straight lines meet at a point they form an *angle.* The straight lines are called *arms* of the angle ; the point, at which they meet, is its *vertex*.

When a straight line stands on another so as to make the adjacent angles equal to one another, each of the angles is called a *right angle* ; and each line is said to

be *perpendicular* to the other. All right angles are equal. A right angle is divided into 90 equal parts, called *degrees* ; each degree, into 60 equal parts, called *minutes* ; each minute, into 60 equal parts, called *seconds.*

An angle, less than a right angle is said to be *acute.* An angle greater than a right angle but less than two right angles, is called an *obtuse angle.*

A *straight angle* is equal to 180° degrees, *i.e.*, two right angles. An angle greater than two right angles but less than four right angles, is called a *reflex angle.*

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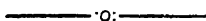
Any portion of a plane surface bounded by one or more lines is called a *plane figure. Rectilinear figures* are bounded by straight lines, called *sides.*

Parallel straight lines lie in the same plane but do not meet, if produced both ways. They always remain at the same distance from each other, though infinitely produced.

A *triangle* is a rectilinear figure with three sides. The angular points are called *vertices.* A side opposite to any vertex, is called the *base* in relation to it.

A *quadrilateral* is a rectilinear figure with four sides.

A *polygon* is a rectilinear figure with more than four sides. If it has five sides, it is called *pentagon* ; if it has six sides, it is called *hexagon* and so on.

A *regular polygon* has *all* its sides and angles equal.

Triangles are distinguished as follows :

The *equilateral* has all its sides equal.

The *isosceles* has two of its sides equal.

The *scalene* has three unequal sides.   '

The *right-angled triangle* has a right angle. The side opposite to the right angle is the *hypotenuse.* Of the two other sides or legs, one is the base and the other is called the *altitude,* or *perpendicular.*

An *obtuse*-angled triangle has an obtuse angle.

An *acute*-angled triangle has three acute angles.

Quadrilaterals are distinguished as follows :

A *parallelogram* has its opposite sides parallel and equal.

A *rectangle* is a parallelogram with all its angles right angles.

A *square* is a rectangle with all its sides equal.

A *rhombus* is a parallelogram having all its sides equal, but its angles are not right angles.

A *trapezoid* has two sides parallel.

A *diagonal* of a quadrilateral is a straight line joining two opposite angular points. Any straight line joining two angular points of a polygon, which are not adjacent, is called a diagonal of the polygon.

A *circle* is a plane figure bounded by one line called the *circumference,* and is such, that all straight lines,

drawn from a certain point within the figure to the circumference, are equal to one another ; this point is called the *centre* of the circle.

A *radius* of a circle is any straight line drawn from the centre to the circumference.

A *diameter* of a circle is any straight line drawn through the centre and terminated both ways by the circumference.

An *arc* is any part of the circumference of a circle.

A *chord* is a straight line which joins the ends of an arc of a circle.

A *segment* is the figure bounded by a chord and the arc of a circle, it cuts off.

A *sector* of a circle is the figure, bounded by two radii and the arc between them. The angle formed by the two radii is called the angle of the sector.

A *zone* of a circle is the portion of the circle contained between two parallel chords.

A sector becomes a *quadrant* when the radii are at right angles. It is thus, a fourth part of the circle. A sixth part of the circle is called the *sextant*, when the angle between the radii is two-thirds of a right angle. When the angle between the radii is half a right angle, the sector is called the *octant*, which is an eighth part of the circle.

A *semi-circle* is the figure bounded by a diameter and one part of the circumference, cut off by the diameter of the circle.

3. Propositions of Geometry.

1. All right angles are equal.

2. The angles which one straight line makes with another straight line upon one side of it, are either two right-angles, or are together equal to two right angles.

3. If two straight lines cut one another, the vertically opposite angles are equal.

4. If a straight line cuts two parallel straight lines, it makes (1) the alternate angles equal to one another ; (2) the exterior angle equal to the interior opposite angle on the same side ; and (3) the two interior angles on the same side, together equal to two right angles.

5. If a side of any triangle be produced, the exterior angle is equal to the sum of two interior opposite angles.

6. The three interior angles of every triangle are together equal to two right angles.

7. All the interior angles of any rectilinear figure, together with four right angles, are equal to twice as many right angles as the figure has sides.

8. In a right-angled isosceles triangle, each acute angle is half a right angle.

9. Each angle of an equilateral triangle is two-thirds of a right angle.

10. If two sides of a triangle are equal, the angles opposite to them are also equal.

11. If two angles of a triangle are equal, the sides opposite to them are also equal.

12. If two sides of one triangle are equal to two sides of another triangle, each to each, and the angle contained by the two sides of the one, equal to the angle contained by the two sides of the other, the triangles are equal in all respects.

13. If two angles of one triangle are equal to two angles of another triangle, each to each, and the side adjacent to the two angles of the one, equal to the side adjacent to the two angles of the other, the triangles are equal in all respects.

14. Parallelograms on the same base, and between the same parallels, are equal to one another.

15. Triangles on the same base, and between the same parallels, are equal to one another.

16. A triangle is equal to half the rectangle having the same base and height.

17. In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the two other sides.

18. The two tangents from any point on a circle are equal.

19. The tangent drawn from any point on the circumference of a circle, is at right angles to the radius from that point.

20. If a diameter bisects a chord of a circle, it cuts it at right angles; and conversely, if it cuts it at right angles, it bisects it.

21.   The angles in the same segment of a circle are equal to one another.   The angle in a semicircle is a right angle.

22.   If a chord be drawn through the point of contact of a tangent, the angle formed is equal to the angle in the alternate segment.

23.   The rectangles contained by the segments of two intersecting chords, are equal.   The same relation-ship holds when the chords meet outside the circle, the rectangles being equal to the square on the tangent from the point of intersection.

.24.   If two triangles be such that the angles of one are equal to the corresponding angles of the other, then the sides opposite to the equal angles, are proportional.

25.   If **ABC** and **DEF** be similar triangles, angles **C** and **F** being corresponding angles and **CG** and **FH** perpendiculars from **C** and **F** on the opposite sides, then **CG : AB : : FH : DE.**

26.   **AB** and **CD** are two chords of a circle meeting at **E.**   If **BC** and **AD** are joined, (1) the triangles **AED, BEC** are similar, for, (2) the angles in the same segment, **EAD, ECB** are equal, and (3) the angles **EDA** and **EBC** are equal.

27.   If **ABC** be a right angled triangle, and **AD** is the perpendicular from the right angle on the hypotenuse, the triangles **DBA** and **DAC** are similar to the triangle **ABC**, and **BD : AD : : AD : DC** or **BD.DC = AD²**.

4. Tables of length, area and volume.

(a) Table of British measures of length.

[Contracted notations are shown in brackets.]

12 inches (in.) make 1 foot.

3 feet (ft.) „ 1 yard.

$5\frac{1}{2}$ yards (yd.) „ 1 pole.

40 poles } or 220 yards } „ 1 furlong.

8 furlongs „ 1 mile.

6 feet „ 1 fathom.

63360 inches, or 5280 feet, or 1760 yards, or 320 poles, or 8 furlongs make a mile. A Gunter's chain consists of 100 links, and is of 22 yards in length (so that each link is about 7·92 inches). There are other chains, *e.g.*, with 1 ft. and $\frac{3}{4}$ ft. links.

(b) Table of British square measures.

144 square inches make 1 square foot. (sq. ft. or ft.²)

9 square feet „ 1 square yard. (sq. yd.)

36 square feet „ 1 square fathom.

$30\frac{1}{4}$ square yards „ 1 square rood or pole.

22 × 22 or 484 square yards make 1 square chain.

40 poles, *i.e.*, 1210 square yards make 1 rood.

4 roods or 4840 square yards make 1 acre.

10 square chains make 1 acre.

(c) Table of Bengali linear measures.

4 cubits make 1 katha.

20 kathas „ 1 bigha.

(d)   Table of Bengali land measures.

1 sq. cubit	make	1	ganda.
20 gandas	,,	1	chattak.
16 chattaks	,,	1	katha.
20 kathas	,,	1	Bigha.
1 bigha	equals	1600	sq. yards.
121 bighas	equal	40	acres.
1936 bighas	,,	1	square mile.
1 katha	equals	8	square yards.
1 chattak	,,	5	sq. yards.

1 acre = 4840 Sq. Yards = 3 Bighas and $\frac{1}{2}$ katha

(e)   Cubic measures (British).

1728 Cubic inches make 1 cubic foot.
27 Cubic feet   ,,   1 cubic yard.

(f)   Table of C. G. S. linear measures.

10 millimetres (mm.) make 1 centimetre.
10 centimetres (cm.) ,, 1 decimetre.
10 decimetres (dm.)   ,, 1 metre.
10 metres   (m.)   ,, 1 decametre
10 decametres (Dm.) ,, 1 hectometre.
10 hectometres (Hm.) ,,   1 kilometre (Km.)

Working in this system is very simple and consists of shifting of decimal points, in changing from one unit to another.   Units of C. G. S. square and cubic measures such as square centimetre, cubic metre, &c., are based on these units and should present no difficulty.

One inch is roughly 2·5 centimetres.

One metre is about 1·1 yard.

One cubic foot of water weighs 62·5 lbs.

One gallon = 10 lbs.

5. Formulae used in Mensuration. The symbols are as in general use.

1. *Square.* When a is side ; d, diagonal :

$$\text{diagonal} = \sqrt{2} \times \text{side} = \sqrt{2}a \; ; \; \text{side} = \frac{\text{diagonal}}{\sqrt{2}}$$

$$\text{area} = a^2 = \frac{d^2}{2} \qquad\qquad \text{p. 82.}$$

2. *Rectangle.* Where d is diagonal : a and b, sides :

$$d = \sqrt{a^2 + b^2} \; ; \quad \text{area} = ab.$$

$$a = \frac{\text{area, or, } a \times b}{b} \; ; \quad b = \frac{\text{area, or, } a \times b}{a} \qquad \text{p.82.}$$

3. *Right-angled triangle.* When h is hypotenuse, a and b, sides :

$$h = \sqrt{a^2 + b^2}$$

$$\text{area} = \frac{ab}{2} = \frac{b\sqrt{h^2 - b^2}}{2} = \frac{hp}{2} \qquad \text{p. 55, 87.}$$

where p is perpendicular on hypotenuse from the right angle.

4. *Triangle.* Where a, b and c are sides $(a + b + c = 2s,$ say$)$:

$$\text{perpendiculars : } P = \frac{2}{a}\sqrt{s(s-a)\,(s-b)\,(s-c)} = \frac{2\text{ area}}{a}$$

$$P' = \frac{2}{b}\sqrt{s(s-a)\,(s-b)\,(s-c)} = \frac{2\text{ area}}{b}$$

$$P'' = \frac{2}{c}\sqrt{s(s-a)\,(s-b)\,(s-c)} = \frac{2\text{ area}}{c}$$

$$\text{p. 64.}$$

$$\text{area} = \tfrac{1}{2}\, a\, P = \tfrac{1}{2}\, b\, P' = \tfrac{1}{2}\, c\, P'' = \sqrt{s(s-a)\,(s-b)\,(s-c)}$$

If medians are p, q and r,

$$\text{area} = \tfrac{1}{3}\,\sqrt{2(p^2q^2 + q^2r^2 + r^2p^2) - (p^4 + q^4 + r^4)} \qquad \text{p. 87.}$$

5. *Equilateral triangle.* Where a is side and h is altitude :   p. 63.

$$h = \frac{a\sqrt{3}}{2}$$

$$\text{area} = a^2 \frac{\sqrt{3}}{4}$$ p. 87.

6. *Isosceles triangle.* Where base is a ; side b :

$$\text{area} = \frac{a}{4} \sqrt{4b^2 - a^2}.$$ p. 87.

7. *Parallelogram.* When a is base, h, height, b and c, other sides :

area $= a \, h$.
$\quad = 2\sqrt{s(s-a)\,(s-b)\,(s-c)}$, if $s = \frac{1}{2}\,(a+b+c)$.   p. 83.

8. *Rhombus.* Where d' and d'' are diagonals :

$$\text{area} = \tfrac{1}{2} d' \, d''.$$ p. 88.

9. *Quadrilateral.* Where d is diagonal, p', p'', perpendiculars from angular points on the diagonal :

$$\text{area} = \tfrac{1}{2} d \, (p' + p'')$$ p. 88.

Area of quadrilateral inscribed in a circle

$$= \sqrt{(s-a)\,(s-b)\,(s-c)\,(s-d)}$$

when a, b, c, d are sides, and

$$s = \frac{a+b+c+d}{2}$$ p. 93.

10. *Trapezoid.* Where h is height, a and b, parallel sides :

$$\text{area} = \tfrac{1}{2} \, h \, (a+b)$$ p. 83.

11. *Regular polygon.* When r and R are radii of inscribed and circumscribed circles.

$$\text{Area} = \frac{nar}{2} = \frac{na}{2} \sqrt{R^2 - \frac{a^2}{2}}$$

a being the length, and n number of sides.   p. 98.

Area of a regular Hexagon $= \frac{3}{2}a^2\sqrt{3}$.

    ,,   ,,   ,, Octagon $= 2a^2(1 + \sqrt{2})$.   p. 98.

    ,,   ,,   ,, Dodecagon $= 6a^2\sqrt{\frac{7}{4}} + \sqrt{3}$.

12.* *Circle.* Where r is radius :

diameter of a circle is $D = 2r = \dfrac{\text{circumference}}{\pi}$ p. 76.

circumference of a circle, $C = 2\pi r = \pi D$.

area of circle $= \pi r^2 = \dfrac{D^2\pi}{4} = \dfrac{C^2}{4\pi}$.   p. 91.

chord $c = 2\sqrt{h(2r - h)}$, where h is height of arc. p. 76

chord of semi-arc $= \sqrt{dh}$ p. 75.

$$\text{Arc (length) } l, = \frac{a}{360} \times 2\pi r,$$

where a is angle subtended at the centre.   p. 77.

Area of circular ring $= (R^2 - r^2)\pi$, where R is outer radius.   p. 93.

Area of ellipse $= \pi\, ab$, where a and b are semi-axes.

13. *Simpson's rule.* When p', p''…are lengths of ordinates, d, common distance :   p. 54, 99.

$$\text{area} = \frac{d}{3}\left[p^1 + p^{2n+1} + 2(p^3 + p^5 \ldots p^{2n-3} + p^{2n-1})\right.$$

$$\left. + 4(p^2 + p^4 + p^6 + \ldots p^{2n-2} + p^{2n})\right]$$

$$= \frac{\text{Common distance}}{3}\ (\text{ first ordinate } + \text{ last ordinate}$$

$+ 2 \times$ sum of odd ordinates $+ 4 \times$ sum of even ordinates.)

14. *Cube.* Where a is an edge :

        surface $= 6 \times a^2$,

        volume $= a \times a \times a = a^3$.   p. 105.

15. *Prism.*
    Surface = perimeter of base × height ;
    volume = area of base × height ;   p. 105.
    volume, if rectangular, = length × breadth × height.

16. *Cylinder.*   Where r is radius, h is height :
    surface $= 2\pi rh + 2\pi r^2$
    volume $= \pi r^2 h$ p. 107.

17. *Cone.* Where l is slant height ; r, radius ; h, height :

    curved surface $= \pi rl = \pi r\sqrt{r^2 + h^2}$.
    ' volume $= \frac{1}{3}\pi r^2 h$.   p. 115.

18. *Pyramid.*
    slant surface $= \frac{1}{2}$ perimeter of base × slant height.
    volume $= \frac{1}{3}$ area of base × height.   p. 114.

19. *Wedge.* Where a and b are length and breadth of base ; h, height, e, edge :
    $$\text{volume} = \frac{hb}{6}(2a + e).$$ p. 116.

20. *Sphere.*
    surface $= 4\pi r^2 = \pi D^2$
    volume $= \frac{1}{3}\pi r^3 = \frac{1}{6}\pi D^3$.   p. 118.

SECTION 2.

Common Instruments and their use. Problems. Lines and angles. Perpendiculars and Parallels.

6. (*a*) It is necessary to have a good instrument box.

Pencils should be of good quality and of hard lead, say **HH.** They should be carefully sharpened like a chisel from both sides, and finally to a sharp edge on fine sand paper. Lines should be thin and drawn lightly.

The *straight edge* or ruler should be bevelled on one side A little practice will indicate how far from any point (through which the pencil is to pass), the edge is to be placed and practice alone will give the habit of keeping the pencil at the same inclination while drawing a line, along a straight edge.

Two *set-squares* are used, each with one angle, a right angle. In one, each of the other angles is 45°. In the other, one angle is 60° and the other, 30°.

The *compasses* should have one fine needle point, and the other, a pencil point. The *divider* has both, needle point· The compass is used to draw circles, by keeping the needle point fixed and taking the pencil point round it.

The *scales* are of constant use for measuring lines and for marking off lengths to numerical data. They can of course, be used like a straight edge in drawing line.

The subdivisions may be in decimals of the inch or centimetre. In the *foot rule*, an inch scale with decimal divisions or divisions in fractions of the inch, is commonly marked. In ordinary practice, the scale is sometimes applied direct to the paper and any required length read off the paper, or marked on the paper, by the pencil. More accurate measurements are however obtained by the use of the divider adjusted (1) on the scale to any desired length, to mark it on paper ; or, (2) on the paper to any given length, to ascertain its value by next placing it on the scale.
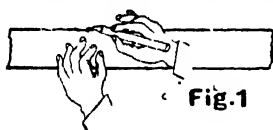
Fig.1

(*b*) In drawing figures to measurement, use is made of scales, set-squares, divider and compasses.*

Let us draw a square of $2\frac{1}{2}$ inch sides and to divide it into squares of $\frac{1}{2}$ in. sides
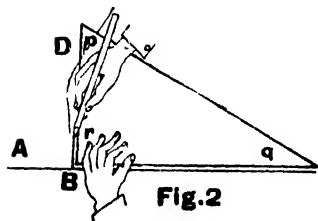
Fig.2

Draw a line by placing the *foot rule* (Fig. 1.) approximately parallel to the foot of the page, (we shall call it horizontal). Let **ABC** (Fig. 2) be this line.

Mark points from **B**, for $2\frac{1}{2}''$ length on it, at every half an inch. Now put the set-square, *pqr*, with one side, *rq*, (not the hypotenuse) along the line **AC**, with the corner, *r*, of the right angle, at the point **B**. Draw a line (we shall call it vertical) along the other side, *rp*,

It is presumed that the students know these instruments.

of the set-square, which should be so placed that
with the proper incli-
nation of the pencil
(which should be
kept the same.), the
line passes exactly
through the point **B**
of the line. In the
same way (with the
set-square reversed),

draw another vertical line at
the other extreme mark. Now
use your foot-rule to mark
these vertical lines at
every half an inch. Draw
lines through these points
parallel to **BC**, also with your
foot-rule. Now place one of
the set-squares with the hy-
potenuse parallel to and a
little below the horizontal
line so that the other set-
square stands on it, with a
side coinciding with one of
the vertical lines. Then keep
the lower set-square in posi-
tion by the fingers of the left
hand and draw lines along
the vertical side of the other,
successively placed along the
marks on the horizontal line and held by the thumb.

Fig. 3

2
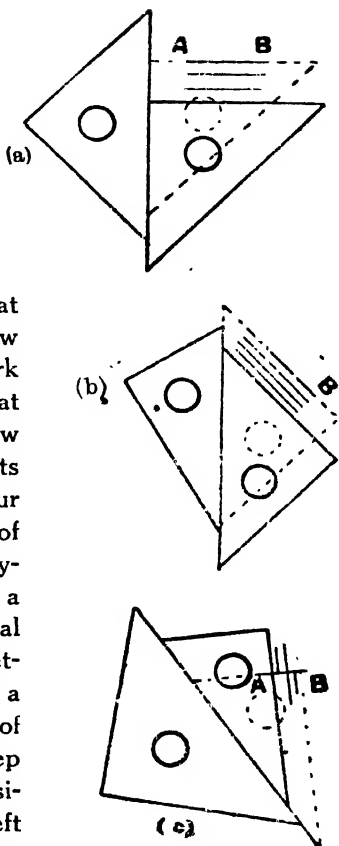
It may be more convenient to turn the paper round [Fig. 3 (a)] so that lines are drawn horizontally. Other ways of drawing parallel lines by 2 set-squares, are shown in Fig. 3 (b) & (c).

EXERCISE 1.

1. Draw a line equal to 6½ inches and divide it into 13 equal parts. Draw squares on these parts alternately above and below the line. Use two set-squares to draw the vertical lines and the foot-rule only twice for all the other lines.

2. Draw a square of 8″ sides and proceed to mark a chessboard on it. By using 2 set-squares hatch alternate squares.

3. Draw a line 5½″ long and divide it into 11 equal parts. Draw one square on the first, two on the second, (one on the top of the other) and so on, until the fourth part, and then in the descending order.

4. Draw a rectangle of 3″ and 2″ sides. Divide it into 6 equal rectangles.

5. Draw a line perpendicular to a given line and draw parallels to the latter at ½″, 1″, 1½″ and 2″ from it.

6. Assuming that 1 square unit is an area formed by 1 unit of length and 1 unit of breadth, prove that 3×4=12.

7. Examples. (a) Draw a regular hexagon and an equilateral triangle in a circle of 1½″ diameter.

Spread out the points of the compasses to ¾″ on the
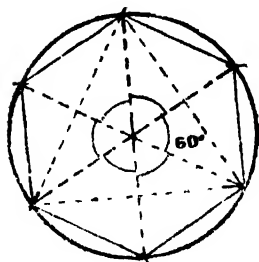
Fig.4

foot-rule and draw a circle (Fig. 4.) with it. Mark along the circumference, lengths equal to the radius. The compasses as opened out or the divider similarly opened out to ¾″, may be used. There will be exactly 6 lengths round the circle. Join the points to form the hexagon.

By joining the alternate points, an equilateral triangle is obtained. The hexagon may also be obtained by placing the 60° set-squares on a diameter with the 60° corner at the centre, and drawing an angle at 60° to the diameter and so on.
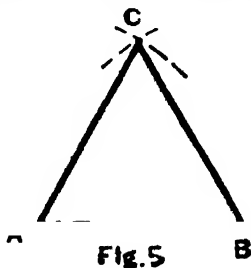
EXERCISE 2.

1. Draw 2 straight lines at right angles. Use the set-squares to divide the 4 right angles into 6, 8, and 12 equal parts.

2. Draw an angle of 15° with set-squares.

3. Draw a right angle and divide it into 6 equal parts.

4. Draw concentric circles of radii, increasing from 1 to 4 inches by half an inch. Inscribe equilateral triangles in these with parallel sides.

5. Draw a parallelogram of 2 and 3 inches sides, one of the angles being 45°.

6. P is a given point and **AB**, a given line. Draw a line through **P**, (a) parallel to **AB**, (b) perpendicular to **AB**, and (c) at an angle of 30° to **AB**, using your set-squares.

7. Use your scale to measure the diagonals in Ex. 5, and lengths of lines from **P** to **AB** in Ex. 6 (b).

————:o:————

(b) Draw an equilateral triangle of $2\frac{1}{8}''$ sides.

Draw a line **AB** (Fig. 5). Spread out your compasses to $2\frac{1}{8}''$ and mark points **A** and **B**, $2\frac{1}{8}''$ apart on **AB**. With **A** and **B** as centres, and radii **AB** and **BA**, draw arcs to meet at **C**. Join **AC** and **BC** by using a straight edge. **ABC** is the required triangle (drawn half size).

Fig. 5

(c)   Construct a triangle of 5, 4, and $4\frac{1}{2}$ ins. sides.
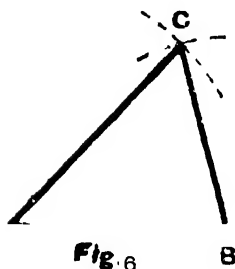
Draw   a   line   **AB**   (Fig. 6) equal to $4\frac{1}{2}''$ and with **A** and **B** as centres and radii equal to 5 and 4 ins.; draw arcs to meet at **C**. Join **AC** and **BC**. ABC is the required triangle (drawn quarter size).

Fig. 6   **B**

EXERCISE 3.

1.   Draw an equilateral triangle of 3″ sides.   Mark the middle points of each of the sides, using the scale.   Draw the medians, they meet at a point.   Join the middle points to form another triangle. This is also equilateral.   The point of intersection of the medians is the centre of the inscribed and the circumscribed circles, of the original triangle.   Show this by drawing them.

2.   Place a chord, 4″ long, in a circle of 3″ radius

3.   Draw an isosceles triangle of 2″ base and 3″ sides.

4.   Draw a parallelogram of 2 and 3 ins. sides and altitude $1\frac{1}{2}''$.

5.   Draw a triangle of $1\frac{1}{2}$, 2 and $2\frac{1}{4}$ ins. sides.

6.   Draw an isosceles triangle on 2″ base and base angles, 45°.

7.   Draw triangles with

(a)   sides 2″ and 3″ and the included angle 45°.

(b)   sides $2\frac{1}{2}$ and $2\frac{1}{4}$ ins. and angle 60°, opposite to $2\frac{1}{4}''$ side.

(c)   two angles 30° and 45° at the ends of a side, 2″ long.

(d)   two angles 60° and 45° and side opposite to the smaller angle, equal to $2\frac{1}{4}''$.

8.   The distance of the point of intersection of the medians of an equilateral triangle from a vertex is 2″.   Construct the triangle.

[Draw a circle of 2″ radius and construct an equilateral triangle with the vertices on it.]

9.   Use your scale to measure (a) the medians and (b) the sides, of the 2nd. triangle in Ex. 1 : (c) the medians in Exs. 3, 5, and 6 ; and (d) the diagonals in Ex. 4.

————:0:————

(d)   Inscribe a square in a circle of 2″ radius.

Draw the circle (Fig. 7) and a diameter by placing the ruler against the centre. Place a set-square with one side along this diameter and the corner with the right angle, at the centre. Draw half of the perpendicular diameter. Complete it with the ruler. Join the extremities to form the square.
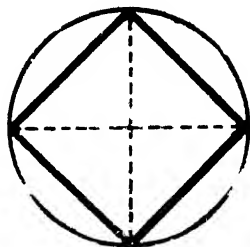

Fig. 7

(e)   Draw a regular octagon inside a circle.

Draw a figure (Fig. 8) as in the last problem and find the middle points of the quarter circles with the help of the 45° set-square. Join to form an octagon.
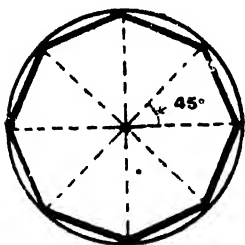
(f)   Construct a square with the length of the line from one corner to the middle point of one of the opposite sides equal to $1\frac{1}{2}$″.

Fig. 8

Construct any square and draw the line from one corner to the middle point of an opposite side. Measure $1\frac{1}{2}$″ along this line from the corner. With the help of two set-squares, draw parallel lines [see § 6 (b)] through this point to complete the square.

EXERCISE 4.

N. B. The method of solving simple problems as above, with the help of the usual instruments, is constantly followed in mechanical drawing. But there are certain constructive methods based on Euclid's elements, accompanied by the use of instruments, which are not only more convenient but are also more accurate. As they are useful in illustrating the applications of geometrical principles, as well as in the training in the practice of the instruments, great importance is attached to them. In the first stages, use of anything more than a ruler and compasses should be avoided as much as possible and the principles of geometry brought to the foreground. Later, when the principles have been learnt, use of instruments, which facilitates the work, should be more and more taken recourse to.

———:o:———.-

1.   Draw a square in a circle of $2\frac{1}{2}''$ radius.

2.   Draw a regular octagon inside a circle of 2″ radius.

3   Construct a square in which the distance from one corner to the middle point of either of the opposite sides is 2″.

4.   Using your scale, find out (a) the side of the square in Ex. 1., (b) the side of the octagon in Ex. 2, (c) the side and diagonal of the square of Ex. 3.

5.   Draw an equilateral triangle of 2 ins. sides ; circumscribe a circle and inscribe a square in it.

6.   Draw a regular hexagon in a circle by using set-squares as in (d), p. 28.

7.   Draw an equilateral triangle in a circle of 2″ radius by using set-squares from the centre.

8.   Draw a regular octagon in a circle of 3″ radius.

9.   Inscribe a figure inside a circle, whose sides will subtend angles of 30°, 45°, 60°, 90°, 45° and 90°, in order, at the centre.

———:o.———

8. Examples. Constructions for lines and angles.

N. B. Dotted lines are used for construction only. Full thin lines show what is given. Full thick lines show what is required.

(*a*) •Bisect a given line, **AB**.

Let **AB** be the given line (Fig. 9). With centres **A** and **B** and any radius (greater than half of **AB**) draw arcs to cut one another at **C** and **D**. Join **CD** to cut **AB** at **E**. Then **AB** is bisected at **E**, by **CD** (perpendicular to **AB**).
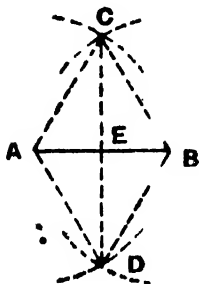
(*b*) Bisect a given arc.

Proceed exactly as above, considering an arc **AB** instead of the line **AB**.

Fig. 9

(*c*) Bisect a given angle.

Let **AOB** (Fig. 10) be the given angle. With centre **O** and any convenient radius, draw an arc cutting **OA** and **OB** in **C** and **D**. With centres **C** and **D** and any radius greater than half of **CD**, draw arcs cutting at **E**. Join **OE**. **OE** then bisects the angle **AOB**.
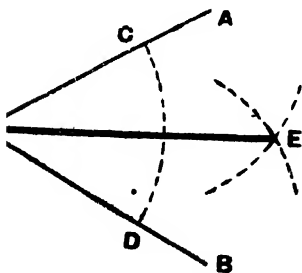
Fig. 10

The process may be repeated for divisions into $2''$ parts, where *n* is any integer. In practice, the

divider will be used, and the points of division of an
arc **CD**, drawn, would be found by trial.

(*d*) Make an angle equal to a given angle **POR**,
on a line **AB** at **A**.

With centre **O** (Fig. 11), and any radius, draw an
arc **MN** to cut **OP**, **OR** at M and N. With the same
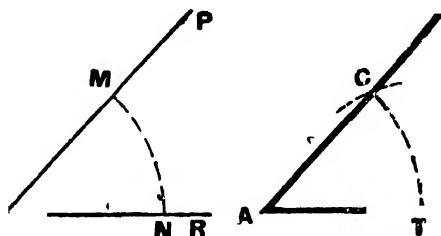radius and centre **A** draw an arc to cut **AB** at T.

Fig. 11

Measure the length **MN** on your compasses and with
centre **T**, draw an arc to cut the previous arc through
T, at **C**. Join **AC**.

The angle **CAB** is the required angle.

N. B. Addition and subtraction of angles can be made on the
same principle. We have simply to draw arcs with the vertices as
centres, of the same radius. The figures are then to be placed on
the same side of a common arm in case of subtraction and on
opposite sides, in case of addition. The difference or sum of the
angles is then easily obtained.

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EXERCISE 5.

Do not use set-squares.

1. Draw a 5″ line. Divide it into 4 equal parts by continued bisection.

2. Draw a circle of 4″ radius. Open the divider to 2, 3, 4 and 5 ins. successively and set off these lengths on the circumference consecutively. Bisect them and show that the bisectors, obtained as above, meet at a point.

3. Draw any line and at a point on it, bisect the two straight angles formed, on its either side

['This, in fact, gives the two perpendiculars at the point]

4. Construct angles of 22°. 30′ and 67° 30′, using only a straight edge and a compass

5. Draw an isosceles triangle with a vertical angle, (a) double and (b) 4 times, a base angle.

[A chord equal to the radius subtends 60° at the centre]

6. Construct a triangle of 2″ base and two adjacent angles equal to 60° and 45°.

7 Construct a triangle of sides 2″ and 3″ and an angle opposite to the 2″ side, equal to 60°.

8. Construct a parallelogram of 2 and 3 ins sides with 45° angle between them.

9. Draw any triangle and verify approximately by adding the angles that the sum of the three angles is equal to 2 right angles. Similarly find the sum of the interior angles of any quadrilateral. Produce the sides in order and find the sum of the *re-entrant* angles.

10. Discuss by actual trial that the radius of arcs in the above problems for bisection, cannot be less than half the line or the arc ; that to give a well•defined point of intersection, they should not be too small or too big (lines at the point would be best at 90°) : that only a small portion of the arc need be drawn for construction ; that in case of bisection of the angle, the bigger the radius, the better is the result.

[Avoid alteration of the compass points, as much as possible.

9. Examples.   Perpendiculars and parallels.

(*a*)   Draw a perpendicular from a point on a given line, when it is not near either end.

Let **AB** (Fig. 12) be the given line and **C** the point on **AB** not near either end.

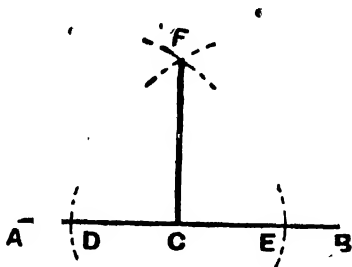With centre **C** and any suitable radius, draw arcs, cutting **AB** at **D** and **E**.

Fig. 12

With **D** and **E**, as centres and any convenient radius draw arcs cutting at **F**.   Join **CF**.
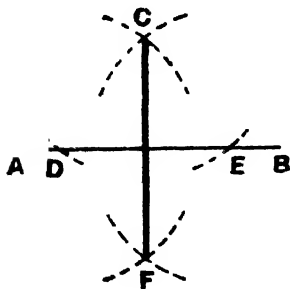
Then **CF** is perpendicular to **AB**.

(*b*) Draw a perpendicular on a line from a point outside it, but not near either end.

Follow the above construction, taking **C** outside **AB** and **F** on the side of **AB**, other than **C** (Fig. 13).

CF is perpendicular to **AB**.

Fig. 13

(c) Draw a perpendicular from a point **C**, on a line **AB**, near or at one extremity.

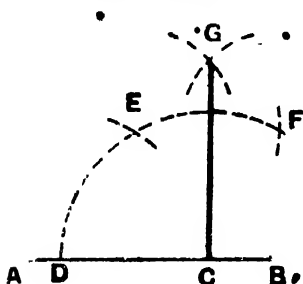With the point **C** (Fig. 14) as centre and any radius, draw a circle to cut **AB** in **D**. Place lengths of this radius along the circle, twice from **D**, setting off **E** and **F**. With centres **E** and **F** and any radius draw arcs to cut at **G**. Join **CG**.

Then **GC** is the required perpendicular.

Fig. 14

(d) Draw a perpendicular from a point **C**, outside a line **AB** and over either end, on the line.

Take any point **D** (Fig. 15) on **AB** and join **CD**. Bisect **CD** in E. With centre **E** and **CE** as radius, draw a circle cutting **AB** in **F**. Join **CF**.

Then **CF** is the required perpendicular.
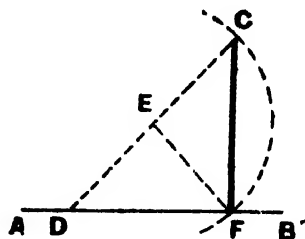
Fig. 15

N. B. In practice, set-squares will be used for the above constructions. One will be placed with an edge coinciding with **AB**, and kept pressed down while the other, with the side adjacent to the right angle, slided on the first, until the vertical edge coincides with **C**, wherever its position may be. The perpendicular line is then drawn along the vertical edge.

EXERCISE 6.

Use only the compasses and the ruler.

1. Construct a square of 2″ sides.

2   Construct a triangle of 2, 3 and 4 ins. sides and draw perpendiculars from the angular points to the opposite sides.

3. Draw capital letters **E, F, H, L, T** of 2 ins. height each

4. Construct a triangle **ABC** with **AB**, **BC** and **CA**, equal to 6″, 4″ and 3″ respectively.   Draw perpendiculars from **A, B** and **C** to **AB** ; and from **A** and **B** to **BC** and **AC** produced.

5. Construct an equilateral triangle of 3 ins. sides   Bisect the sides and erect perpendiculars at the points of bisection.   Do they meet ?

–:o:–

(*e*) Draw a line parallel to a given line at a given distance from it.

Let **AB** (Fig. 16) be the line. Draw two perpendiculars **AD, BC** from any two points **A,B**, on the line, as far apart as possibe.
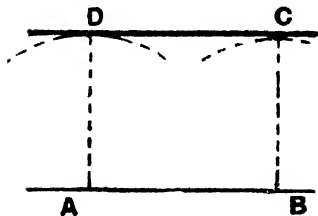

Fig. 16

Cut off lengths **AD** and **BC** equal to the given distance.

Join . **CD**. Then **CD** is the required parallel.

Or, instead of drawing **BC** perpendicular to **BA**, we can draw **DC**, perpendicular to **AD** from D.

(*f*) Draw a line parallel to a given line through a given point (outside it).

Let **C** be the given point (Fig. 16) and **AB** the given line.

After drawing **CB** perpendicular to **AB**, either draw second perpendicular **AD** equal to **BC**, and proceed above.

Or, simply draw **CD**, perpendicular to **BC** at **C**.

Alternatively, take any point **E** on **AB** (Fig. 17) and with centre at this point, draw a circle through **C** which cuts **AB** in **F**.

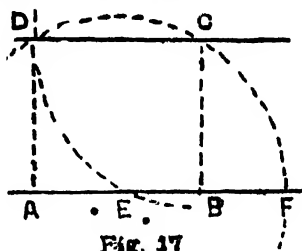With centre **C** and same radius draw an arc cutting the circle in **D**. Join **CD**. Then **CD** is parallel to **AB**.

Fig. 17

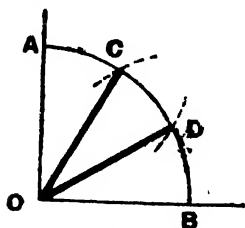In practice, two set-squares will be used to draw the parallel, as indicated.

Fig. 18

(g) Trisect a given right angle.

Draw the right angle **AOB** (Fig. 18) and with centre **O** and any radius draw a quarter circle cutting **OA** and **OB** at **A** and **B**.

With centres **A** and **B** and the same radius, mark points **C** and **D** on the arc.

Join **OC** and **OD**.

The right angle **AOB** is trisected at **C** and **D** by **CO** and **DO**.

EXERCISE 7.

Use only the compasses and the ruler.

1. Draw a rectangle of 2 and 3 ins. sides.

2. Draw a triangle of 2, 3 and 4 ins. sides. Draw from the vertices, parallels to the opposite sides.

3. Draw a parallelogram of 1½″ altitude, 2″ base and one angle, 30°.

4. Draw the figures of

(a) **X** and **Z** with two-angles equal to 30°.

(b) **H** with the horizontal line equal to 1 ins.

(c) **V** and convert it into **M** by drawing vertical lines.

10. The protractor and its use.

(a) The *protractor*. Angles of 90°, 60°, 45°, 30°, 15°, 75°, are easily drawn by the set-squares or by construction. Protractors are constructed by an extension of the method of construction, and allow *any* angle
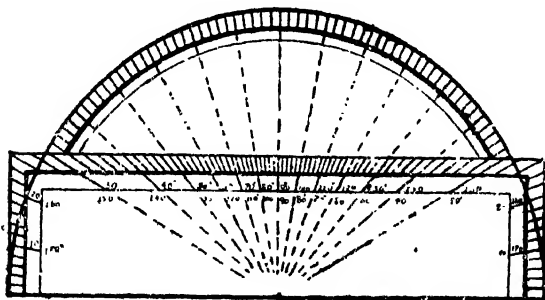

Fig. 19

to be set off. A semicircle (Fig. 19) on 6 ins. diameter, is drawn on a plate, and divided into 180 equal parts.

When these divisions are joined to the centre at the base of the semicircle, the angle at the centre (i. e. 2 right-angles) is divided into 180 equal parts, called *degrees*, which are marked along the perimeter of the semicircular plate. The plate may also be rectangular in form, as is also shown in the figure. The plate is placed with the base along the line on paper, on which the angle is to be drawn, with the centre at the point from which the angle is to be drawn. The point, marked on paper, against the required angle at the perimeter, is joined to the given point and makes the required angle, with the line.

(*b*) *Inscribe* a regular pentagon in a circle.

The angle subtended at the centre by any side is equal to 360°/5, that is, 72°.

Draw the circle and 5 radii, each inclined at 72°, to two others.

Join the extremities to form the pentagon.
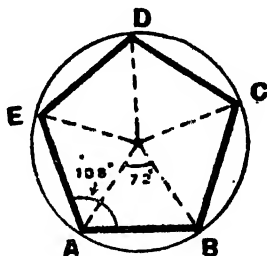
(*c*) Construct a regular pentagon on a given line.

Fig. 20

On the given line **AB** (Fig. 20), plot angles **BAE** and **ABC**, each equal to* (5 × 2 × 90° − 4 × 90°)/5, *i. e.*, 108°.

Cut off **AE** and **BC**, each equal to **AB** ; draw lines from **E** and **C**, each equal to **AB**, meeting at **D**.

Then **ABCDE** is the required pentagon.

$\frac{2(n-2)}{}$ rt. angle, where *n* is the number of sides.

EXERCISE 8.

1. Inscribe a regular heptagon in a circle of 2″ radius. Also construct one on a 2 ins. line. Proceed, with angles calculated as above

2. Draw lines meeting at a point at 18°, 70°, 100°, 108°, 110°, 135°, 150° and 170°.

3. Construct a regular decagon (10 sides), a do-decagon (12 sides), and a quint-decagon (15 sides) on 2″ base.

4. Inscribe a regular do-decagon in a circle of 3″ radius.

5. In an angle **AOB**, (**AO**=2″) draw **BA** and **AN** perpendiculars to **OA** and **OB** respectively. Also draw arc **AM**, with centre **O** and radius **OA**, to cut **OB** in **M**. Join **AM**. Measure the angles **AOB**, if drawn so that

     (a) **AM** - 1·5″, 1·2′ and 2·8″.

     (b) **AB**=1·4″, 2″, and 8″.

     (c) **AN**=0·5′, 1″ and 1·8″.

6. Construct a regular pentagon in a circle of 2 ins. radius.

7. Construct a regular hexagon of 2 ins. sides.

8. Inscribe a regular octagon in a circle of 3 ins. radius.

9. Construct any triangle and measure the angles by the protractor. What is the sum ?

10. Draw any quadrilateral and measure the angles by the protractor. What is the sum ?

11. Produce the side of a pentagon and a heptagon in order, and measure the re-entrant angles. What is the sum ?

12. Find by measurements with a protractor, the sum of the (1) angles at the centre, (2) angles at corners, (3) re-entrant angles on sides produced, of any regular decagon, do-decagon and quint-decagon, drawn by you.

—————:o:—————

PLANE PRACTICAL GEOMETRY.

Subdivision of Lines. Scales. Construction of Figures.

11. (*a*) Constructions for division of lines have, so far, been confined to number of divisions, which is some integral power of 2. But other sub-divisions are often required The common practical method is by trial with the divider. The points are opened out approximately to the length of sub-division required, by mere guess, which practice makes accurate. Starting from one end, the length is successively stepped off along the line If out at the other end, the length is changed and another trial given, until the correct sub-division is obtained. Similar construction is also used for arcs. The geometrical construction is given below.

(*b*) Divide a given line **AB** into *n* equal parts.

Draw a line **AC** (Fig. 21) at small angle to **AB**. Take a small length **AP** on **AC** and mark successive
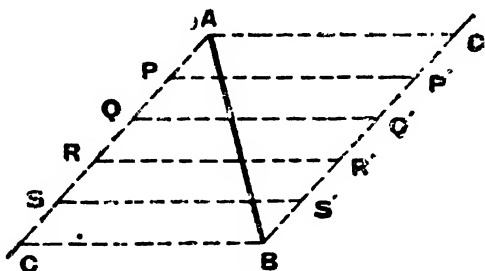

Fig. 21

lengths **PQ, QR** &c., each equal to **AP**, until *n* divisions are obtained at some point **C**. Join **BC**. Draw lines through **P, Q, R** &c., parallel to **CB** (with two set-squares). Then **AB** is divided into *n* equal parts by these lines.

Alternatively, a line **BD** may be drawn parallel to **CA** and lengths are also marked on this, equal to **AP**, at **S', R', Q'** &c. **PP', QQ', RR'** &c. are then joined. These divide **AB** as required. No set-squares are required in this case.

(c) *Scales.* The size of objects to be drawn is generally too large to be drawn in paper, full size. The ratio, called the *representative fraction,* in which the actual dimensions are changed in the representation, depends on the size of the object, space available for drawing and other considerations.

Thus if a length of 24ft. have to be represented in a space of 6 ins, our scale is $1'' = 4'$, or 1/48 full size. If further, we want our scale to read every $\frac{1}{4}'$ or 3'', of the actual size, we proceed as follows :—

Draw (Fig. 22) an oblong $\frac{1}{4}''$ (vertical) by $2\frac{1}{2}''$ (say ; our scale would read up to 10 ft. as a maximum, at one time ; scales are however generally made longer). Divide it by vertical parallel lines, $\frac{1}{4}''$ apart. Each of these divisions then represents 1' of actual size. Draw vertical lines in the first square, every $\frac{1}{16}''$, the third one to rise a little higher. These divisions represent every
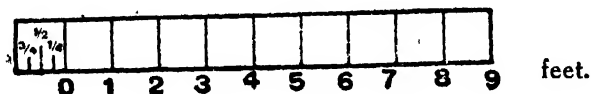
<p style="text-align:center;">Scale $1'' = 4'$.</p>

<p style="text-align:center;">feet.</p>

<p style="text-align:center;">Fig. 22</p>

$\frac{1}{4}'$, or 3'' of actual size. The first $\frac{1}{4}''$ line is to be marked "zero" and 1, 2, 3 &c. "feet" shown to the right, and $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ ft., to the left. These are shown in the figure.

To read any length by this scale, the whole number of feet is read by divisions to the right and fractions, by the space to the left, of the zero line.

This is an example of *reducing scale,* where the ratio, is smaller than unity but in case of an *enlarging scale,* sometimes required, it will be greater than unity.

(*d*) *Diagonal scales.* These are used for dealing with lengths involving small fractions of an inch, with accuracy. Let an oblong **ABCD,** 1″ by 1″ [shown half size in (1), Fig. 23], be divided into 10 equal parts by horizontal parallel lines. Let a diagonal **AC** cut these lines. The distance of the point of intersection of the diagonal
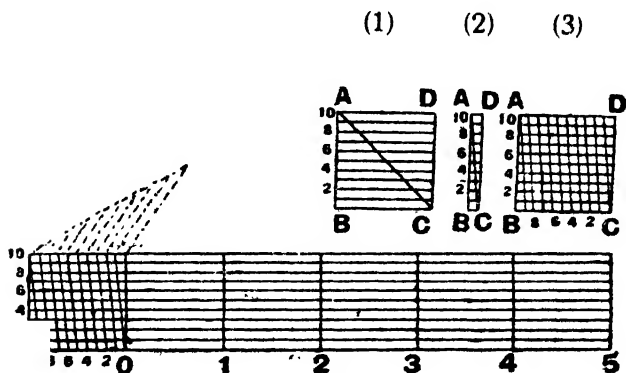

Fig. 23

with the third (say) line from **BC,** from **CD,** is $3 \times 1/10''$ or, 0 3″ and so on, since in successive lines, the point of intersection of diagonal is 0·1″ further away from

BC or CD. If the oblong had **AB** equal to 1″ and **BC,**
0·1″ [as in (2), Fig. 23] ; the distance of the point of
intersection of the diagonal with the 7th (say) line from
CD is $7 \times 1/100$, or, 0·07″ and so on, argued in the
same way. The figure in (3) 23, is the combination of
(1) and (2) 23.

The diagonal scales based on this principle are
marked as shown in figure 23, bottom figure, the inch
divisions, to the right of the zero line, and the tenths
to the left, horizontally. The hundredths are marked
vertically upwards at the left extremity. The figure
gives actual construction also, for drawing the parallel
vertical lines to the left. Similar construction for hori-
zontal parallel lines may be made to the right of the
figure.

Lengths are only taken off such scales by dividers
or measured from adjusted dividers by such scales,
along any one horizontal line. The divisions to the
right of the zero line gives the number of full units ;
horizontal divisions to the left, tenths ; and the vertical
divisions, the hundredths.

Instead of inch divisions and its decimals, any other
scale can be made in the same way and smaller units
may be read direct. For example, if the above were to
represent a scale of 1″ equal to 1 yard, **BC** would
be divided into 3 parts and **AB** into 12, giving feet
and inches, and the scale constructed accordingly.
Readings would be in yards, feet and inches in this
case.

12. Examples. (*a*) Construct a diagonal scale to read inches, tenths and hundredths of an inch.

Draw a rectangle **ABCD** seven inches long (part is shown in fig. 24) and one inch wide ; divide it into seven equal parts ; at the end of the first division from A, fix the zero (0) point and to its right, fix 1, 2, 3, 4, 5, 6
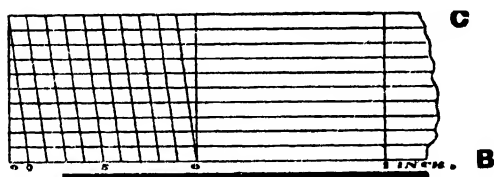

Fig. 24

in each division. Divide **AD**, the vertical line, into ten equal parts and mark them with figures from **A** to **D** and draw lines through these points parallel to **AB**. Divide 0A into ten equal parts (01,...89, 9A) from 0 to **A**. Join D9 ; from other divisions between **A** and 0, draw lines parallel to 9D.

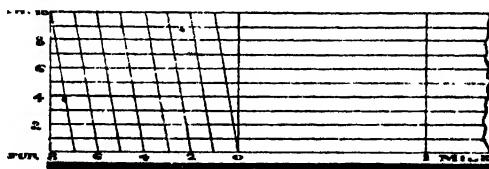(*b*) Construct a scale, **R. F,** = 1/63360, showing Miles, Furlongs and Chains.

Fig. 25

By calculation, it can be shown that 1″ represents 1 mile (1″/63360 in. = 1″/5280 ft. = 1″/1760 yds. = 1″/1 mile.)

Draw a line (Fig. 25) seven inches long. Mark on it at every inch ; at the first division from left, mark the 0 point ; number the primary divisions towards the right— 1, 2, 3, 4, 5, 6 miles ; divide the division to the left of 0, into eight equal parts, that will represent furlongs. Now draw a rectangle on the 7″ line and divide the vertical lines into ten equal parts. Draw lines through each of these divisions parallel to the base line. Like the last scale, complete the diagonals. The primary divisions represent miles ; the left hand divisions divided into eight equal parts, furlongs ; and vertical divisions, chains.

(c) The scale of a map of France is in French leagues. It is found by measuring the scale that 3·75 inches represent 25 leagues. Construct the corresponding scale of English miles. (1 French league = 4262·84 English yards.)

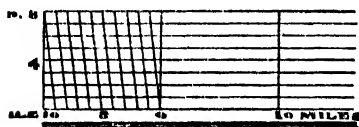Here 25 leagues = 4262·84 × 25/1760 = 60·5 miles.

Fig. 26

Consequently 60·5 miles are represented by 3·75″ ; so the scale may show 110 miles without being very long, as 60·5 : 110 :: 3·75 : 6·81. Divide then, a line 6·81 inches long (Fig. 26) into 11 equal parts to show spaces of 10 miles ; subdivide the first primary division into ten equal parts to show miles. Diagonally dividing, furlongs are also shown vertically.

(d) A map is 36″ long and 24″ broad. This repre-

sents an area of 20 acres.   Draw the scale of the map to
show poles, yards and feet.

Here 36″ × 24″ represent 20 acres, or 864 sqr. inches
represent 96800 sqr.
yards; hence .180 sqr.
inches represent 12100
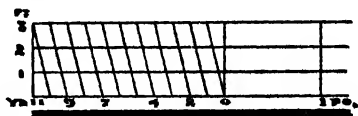sqr. yards, or, 5˙19″ =
55 yards.   Assume

Fig. 27

the length of the scale to be 5˙19″.   Divide the length
5˙19″ into 10 equal parts (Fig. 27).   Each part is a pole.
Divide 2 poles, i. e., 11 yards, into 11 equal parts to show
1 yard.   The zero point of the scale is, in this case, the
second point from the left.   Divide vertically into 3
points, to show feet.

EXERCISE 9.

1   Trisect an angle of 40°, as set off by the protractor, by trial
with the divider.

2. Draw a circle of 2″ radius and a diameter. Bisect one semi-
circle by construction and the other by trial by divider. Join the
points by a line which should pass through the centre and note the
accuracy of your drawing.

3. Draw two straight lines at right angles to the another and
trisect one right angle by using the protractor, next by construction,
and two others by trial. See the accuracy of your figure by noting
corresponding lines of trisection, which should be colinear through the
point of intersection of the 2 st. lines.

4. Divide a line of 4″, into two parts, one of which is double
of the other.

5. Divide a line of 6″ into three parts, in the ratio of 2 : 3 : 4.

6. Divide a line of 4″ into 5 and 7 equal parts.

7. Divide a line of 2″ into 10 equal parts.

8. Construct a scale 4″ long, to read yards and feet, the representative fraction being 1/36.

9. Construct a scale, 1/20 full size, 4″ long, showing feet and inches.

10. Construct the following scales :

Scale.	Long enough for :	Showing.
(a) 1½″ = 1′	6′	ft. and ins.
(b) 1/16	8′	,, ,,
(c) 1″ = 1 yd.	10 yds.	yds. & ft.

11. Draw the plan of a room 16′ by 24′ on a sheet 9″ by 16″ leaving a decent margin all round. Draw and mark the scale that should be used.

12. Draw a diagonal scale (1) 6″ long, in inch, 0 1″, and 0 01″, divisions (2) 4″ long, showing yards, feet and inches, 1/36 full size.

13. Construction of figures to measurement.

(a) Construct a triangle of perimeter 8″, base 2″, and one base angle, 45°.
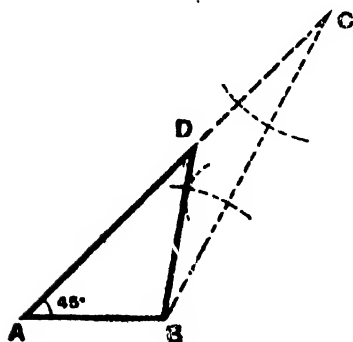

Draw (Fig. 28) the base **AB**, 2′ long and the angle **BAC** equal to 45°. Cut off **AC** equal to 6″ (8″—2″). Join **BC**. Make the angle **CBD** equal to the angle **DCB**. Then **ABD** is the required triangle. (Fig. half size.)

Fig. 28

(b) Construct a triangle of perimeter 8″, altitude 2″ and one base angle, 45°.

Draw (Fig. 29) lines **AB** and **CD**, 2″ apart, parallel to one another. At **A** make angle **BAC**, equal to 45°,
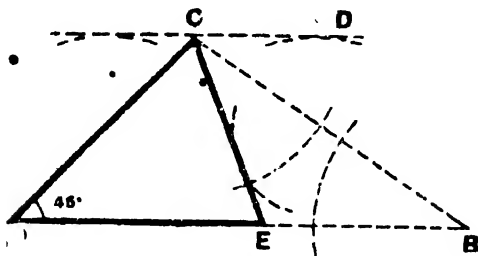

Fig. 29

cutting **CD** at **C**. Take a length **AB**, equal to 8″ minus **AC**. Join **BC**. Make an angle **BCE** equal to angle **CBA**. Then **CAE** is the required triangle (scale—half size).

EXERCISE 10.

1. Construct a triangle with two angles equal to 60° and 45° and a side equal to 2″, (1) adjacent to both, (2) opposite to one of them

[The third angle is 180° - 60° - 45°, and may be obtained by drawing the two angles successively at a point in any line. Then (2) reduces to (1)].

2. Find a point **D** in **EF**, so that the sum of the lines **DA** and **BD** from two given points **A** and **B**, 3″ apart and 2″ from **EF** on the same side of **EF**, may be the least. Measure the distance of **D** from **A** and **B**.

[Draw **AEA′**, perpendicular to **EF**, making **EA′** equal to **EA**. Join **A′B** cutting **EF** in **D**. Join **AD**. Prove by joining any other point **G** on **EF** to **A** and **B** that **AD+BD** is less than **AG+BG**.]

3. Construct a triangle of 4″ base, 3″ altitude and 2″ radius of the circumscribed circle.

4. Construct a triangle of two sides equal to **2** and 3 ins. and (1) the included angle equal to 45°, (2) an angle opposite to 2″ side equal to 45°.

[Set off **AB** equal to 3″ on any angle **CAB** = 45° and with centre **B**, radius 2′, cut off **AC** at **D** and **D′**. Two triangles are obtained for (2).]

5. Construct a triangle of 6″ perimeter with two angles equal to 60° and 45°.

[Make the angles **BAC** and **ABC** equal to 60° and 45° at the ends of the 6″ line **AB**; bisect them to meet at **D** and from **D** draw parallels to **CA** and **CB**].

6. Construct all possible triangles with two angles equal to 60° and 30° and a side equal to 3″.

7. Construct all possible triangles with two sides equal to 2½ and 3½ in. and an angle equal to 30°.

8. Construct a triangle of 8″ perimeter with two angles equal to 90° and 30°.

(c) Construct a rectangle and a square, equal to a triangle of 2, 2½ and 3 ins. sides.

Fig. 30
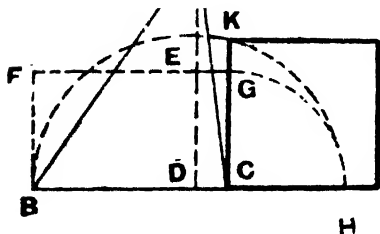
Draw the triangle **ABC** (Fig. 30) to dimensions. Draw the perpendicular **AD** from the vertex **A**, to the base **BC**. Bisect **AD** in **E** and through **E** draw **FEG** parallel to **BC**. Draw **BF** and **CG** from **B** and **C**, perpendicular to **BC**, to meet **FEG** in **F**

and **G.** Then **FC** is a rectangle equal in area to the triangle. (Drawn half size).

This may also be obtained by bisecting (Fig. 31) **AC** in **M** and completing the parallelogram **BM** and then the rectangle **FC**.
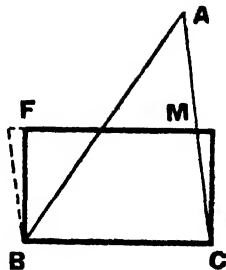

Fig. 31

Produce (Fig. 30) **BC** to **H** making **CH** equal to **CG.** Draw a semicircle on **BH** and produce **CG** to meet it at **K.** A square drawn on **CK,** will be equal to the triangle.

The following may be noted :

If **GC** = 1″ and **FG** = 2″,

the rectangle **FC** = 2 sq. ins. = square on **KC.**

Hence **KC** = $\sqrt{2}$ *inches.*

(*d*) Construct a rectangle on a given line, equal in area to a given rectangle.

Fig. 32
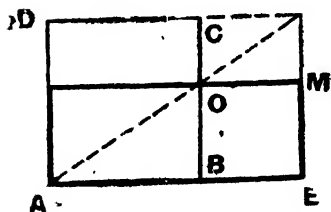
Let **ABCD** (Fig. 32) be the given rectangle. Produce **AB** to **E,** making **AE** equal to the given line. Complete the rectangle **AEFD** and join **AF,** cutting **BC** at **O.** Draw **LOM** parallel to **ABE,** cutting **AD** and **EF** in **L** and **M.** Then **LE** is the required rectangle.

(e) Construct a regular polygon (say, a pentagon) on a line of $1\frac{1}{4}''$ in length.

Let **AB** (Fig. 33) be the given line $1\frac{1}{4}''$ long ; produce it to **C** making **BC = AB**. On **AC** describe
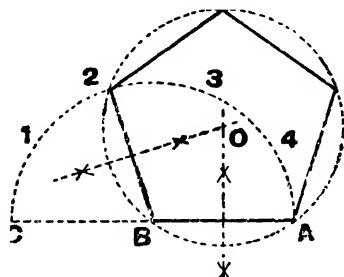

a semicircle ;, divide this into five equal parts (as many as the number of sides of the required polygon) ; join **B2**, bisect **B2** and **BA** and let the perpendicular bisectors intersect at **O**. With **O** as centre, **OB** or **OA** as radius, describe the

Fig. 33

circle ; **BA** and **B2**, chords of the circle, will be two sides of the polygon ; place three other chords in the circle making the pentagon complete.

(f) A trapezium **ABCD** (Fig. 34) is given, having adjacent pairs of sides **AB, BC** equal : inscribe a square in it.

Draw the diagonals ; at one end of a diagonal set off a perpendicular **AE**, equal to the diagonal **AC**. Join **EB** cutting **AD** in **K**. From **K** draw **KM** and **KN** parallel and perpendicular to the dia-

Fig 34

gonals, cutting **AB** and **CD** in M and N. Through M and N draw **MP** and **NP**, parallel to **KM** and **KN** cutting **BC** in **P** ; **KMPN** is the square.
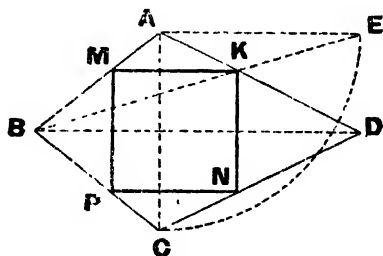
(*g*) Inscribe a square in a given rhombus.
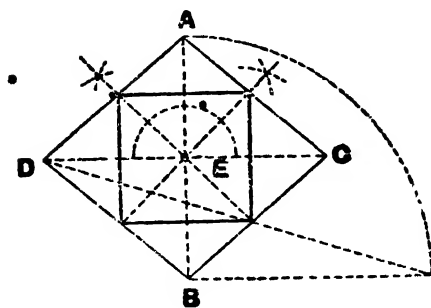
Fig. 35

Draw two diagonals (Fig. 35)· **AB** and **CD** ; bisect the angles **AEC** and **AED** and produce the bisectors till the sides of the rhombus are cut. Join these points and the required square will be drawn. Alternatively, the construction above may be used as shown.

EXERCISE 11.

1. Construct a parallelogram equal to an equilateral triangle of 2″ sides and having an angle equal to 45°, (1) of half the altitude, (2) on half the base of the triangle.

2. Construct a square equal to a triangle of 2½, 3 and 3½ ins. sides.

3. Construct a rectangle equal to another of 2 and 4 ins. sides, having one side equal to 3″.

4. Construct a parallelogram equal to a triangle of 2, 3 and 4 ins. sides, having one side equal to 2″ and an angle, 45°.

5. Draw a square of 1" sides and a rectangle of 1" by 2" sides. Obtain the square roots of 2 and 5 by measuring the diagonals by the divider and scale.

6. Construct 4 rectangles of 1 by 2, 3, 5 & 7 ins. sides respectively and draw squares equal to them. Obtain the square roots of 2, 3, 5 and 7 by measuring the sides as above. Also obtain the square roots of 5 and 10 from the diagonals of the first 2 rectangles.

7. Draw a square of 1" sides. At the far end of a diagonal, erect perpendicular equal to 1" and complete the right angled triangle. Erect a perpendicular equal to 1" at the far end of the hypotenuse and complete the second right-angled triangle. Proceeding in this way, measure the hypotenuses to find the square roots of 2, 3, 4, &c. up to 10.

8 Divide the product of 3 and 4 by 5, by geometrical construction only.

————:o:————

14. (*a*) Reduce the number of sides of any figure, by one, without altering the area.

Let **AB, BC, CD** (Fig. 36) be 3 contiguous sides of

Fig. 36

any figure. Join **AC** and from **B** draw **BM** parallel to **AC**, to meet **DC** produced in **M**. Join **AM**. Then the triangles **AMC** and **ABC** are equal. Three sides **AB**, **BC** and **CD** of the figure have now been reduced to two, **AM** and **MD**, without altering the area.
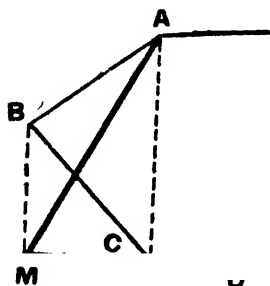
For any re-entrant portion (Fig. 37), such as **GEF**, join **GF** and draw **EN** parallel to **GF**. Join **FN** cutting **EG** at **O**. Then the triangles **ENF** and **ENG** are equal. Hence &c.

The principle is to cut parts off the figure in some places, while replacing equal portions in other places. By continuing the process we can always reduce any rectilinear figure to a triangle.

Fig. 37

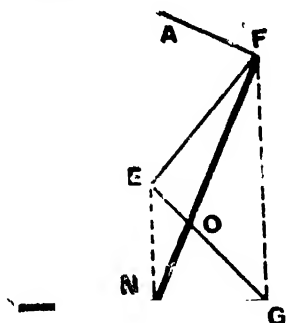(b) Construct a triangle having a given altitude, and equal in area to another.

Let **ABC** (Fig. 38) be the given triangle. Draw **DE** parallel to **BC** at a distance equal to the given
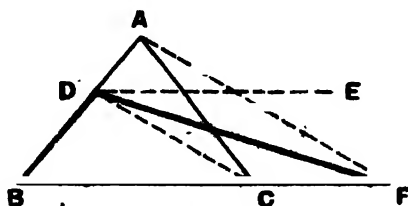

Fig. 38

altitude, cutting **AB** in **D**. Join **DC** and draw **AF** parallel to it, cutting **BC** produced at **F**. Join **DF**. Then **DBF** is the required triangle.

This problem is useful in the addition and subtrac-

tion of triangles ; for, when converted to equal altitudes, such problems simply reduce to the question of addition and subtraction of bases.

(c) Divide a quadrilateral **ABCD** into two equal parts by a straight line drawn from an angular point **D**.
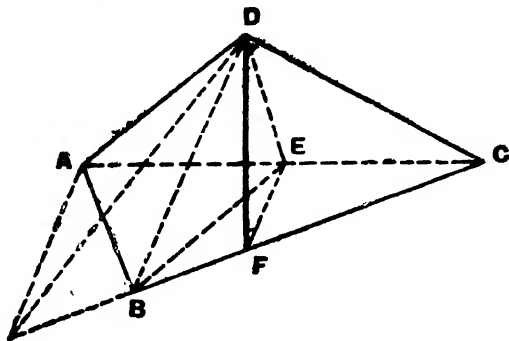
Fig. 39

Join **AC** (Fig. 39) and bisect it at **E**. Join **DE** and **BE**. Each of the figures **ADEB** and **CDEB** is half the quadrilateral. Join **DB** and from **E** draw **EF** parallel to **DB**, cutting **BC** in **F**. Join **DF**. Then **DF** bisects the quadrilateral. Alternatively, from **A** draw **AK**, parallel to **DB**, to meet **CB** produced in **K**. Join **DK**. Triangle **DKC** is equal to the quadrilateral. Now bisect **KC** in **F** and join **DF**. The triangle **DFC** is half the quadrilateral.

EXERCISE 12.

1. Construct a triangle equal to the sum and difference of two triangles of 2″, 3″, 4″ and 2½″, 3″, 3½″ sides.

2. Construct a triangle equal to a parallelogram of 4″ and 5″ sides, having an angle equal to 30°.

3. Divide a quadrilateral of 3″, 3½″, 4″ and 5″ sides into two equal parts by lines through the angular points.

4. Draw a triangle equal to the sum of the parallelogram in (2) and each of the triangles in (1).

5. Construct a parallelogram equal to a quadrilateral of 2″, 3″, 4″ and 5″ sides, having an angle equal to 45°, between 2 and 3 ins. sides, on the 3 ins. side.

6. Draw a regular hexagon in a circle of 2″ radius and join the centre to each of the angular points. Construct a triangle equal in area to the figure, when one of the 6 smaller triangles formed, is cut out.

(d) Divide a triangle into two equal parts by lines drawn from a point in one of the sides.

Let **ABC** (Fig. 40) be the triangle and **P**, the given point on the side **AC**. Bisect **AC** at **D**. Join **BD** and **BP**. From **D** draw **DE**, parallel to **PB**. Join **PE**.

Then **PE** bisects the triangle.

By dividing **AC** into any number of equal parts and extending the

Fig. 40

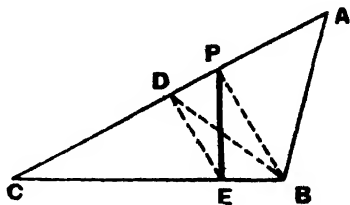construction, the triangle may be divided into the same number of equal parts.

(e) Divide a given triangle into two equal parts by a line parallel to the base.

4

Let **ABC** (Fig. 41) be the given triangle, **BC** being the base. Describe a semicircle on one of the sides, say on **AB** and bisect it at **D**. With centre **A** and radius **AD** describe an arc cutting **AB** in **E**. From **E** draw **EF** parallel to **BC**. Then **EF** bisects the triangle.
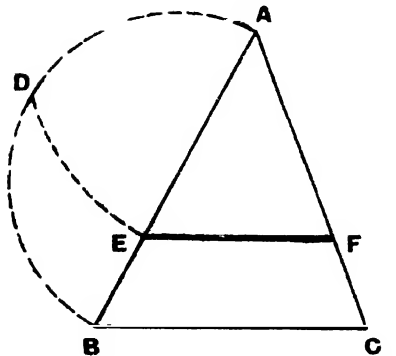
Fig. 41

[**AE = AD = AB**/$\sqrt{2}$ and areas of similar triangles are as the squares of the sides.]

EXERCISE 13.

1. Bisect and trisect a triangle of 2, 3, and 4 ins. sides, by lines from its angular points and also from the middle point of its 4 ins. side.

2. Bisect a parallelogram, 3″ and 4″ sides, containing 45°, by lines (1) through the angular points ; (2) through the middle points of the sides and (3) through any given point, elsewhere.

3. Bisect a parallelogram, 3×4 ins. sides at 45°, by (a) a line perpendicular to the 3 ins. side, (b) a line parallel to the 4 ins. side, and (c) lines perpendicular to the diagonals.

4. Bisect a triangle of 2″, 3″ and 4″ sides by a line parallel to the base.

———:o:———

(*f*) Construct a square with its diagonals equal to 3 ins.

Draw a circle of radius equal to $1\frac{1}{2}″$. Draw two diameters at right angles to one another and join their extremeties to form the square.

(*g*) Construct a right-angled triangle of 3 ins. hypotenuse and one side equal to $2\frac{1}{4}$ inches.

Draw a semicircle of $1\frac{1}{2}''$ radius on hypotenuse **AB**. With **A** as centre and radius equal to $2\frac{1}{4}''$, mark **C** on the semicircle. Join **AC** and **BC**. **ABC** is the required triangle.

Or, erect a perpendicular on one extremity of a $2\frac{1}{4}''$ line and with the other extremity as the centre and radius equal to 3", mark on this perpendicular.

(*h*) Draw a tangent to a circle from a given point.

When the point is on the circle, simply draw the radius through the point and erect a perpendicular at its end.

When the point is outside, join it to the centre and erect a semicircle on this line. Join the given point to the points of intersection of this semicircle on the circle. These are the required tangents.

To draw common tangents to 2 circles, (1) direct, or (2) transverse ; draw circles with radius equal to (1) difference, or (2) sum, of the 2 radii, concentric with the bigger circle. Draw tangents to this circle from the centre of the smaller circle, as above. The line through the centre of the bigger circle and the points of contact of the tangents gives on the bigger circle, the points of contact of (1) direct, or (2) transverse tangents. Tangents from these points to the smaller circle, are the required tangents.

In practice, set-squares and rulers would be used. There would not be the same accuracy, but a fair amount is obtained by a practised hand.

(*i*) Describe a circle to pass through (1) two given
points **A** and **B** and to touch a given line **CD** ;
(2) three given points.

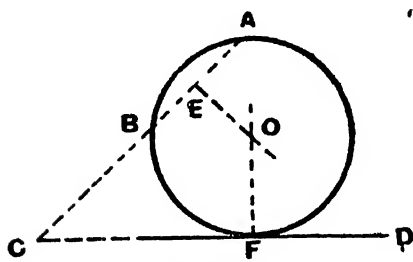(1)   Join **AB** (Fig. 42), bisect it at **E** and erect **EO**
perpendicular   to
AB.   The   centre
of   the   required
circle must lie on
EO.

Let **AB** pro-
duced meet **DC** at
C.   Find   a   point
F in **CD**, so that
$CF^2 = CB . CA$.

Fig. 42

The required circle passes through **A**, **B** and **F**.

The centre is at **O**, where the perpendiculars **EO** and
OF (to **AC** and **DC**), intersect.

(2)   Construction follows from first part of (1)

<div style="text-align:center">**EXERCISE 14.**</div>

1.   Construct a rectangle on a line of 3″ as diagonal, having a
side equal to 2 inches.

2.   Use your set-square (45° only) to obtain points of a semicircle
on a given line **AB**.

3.   Construct a right-angled triangle on a line ·of 2 ins. (hypo-
tenuse), having a side equal to 1¾ inches.

4.   Construct a square on a line of 2″ diagonal.

5.   Solve the following right-angled triangles, i. e. find the other
sides and angles :

(1)   Hypotenuse, 3″ ; a side, 2″. (2) Hyp. 3″ ; height, 1″.
(3)   Hyp. 3″, an angle, 60°.   (4) Hyp. 2¼″, a side double of another.

(5) Hyp. $3\frac{1}{4}''$, one angle double of another. (6) Height $1''$, one base angle $30°$. (7) A side $2''$, a base angle, $45°$.

·   6. A tin roof rises one in four. Find the floor length covered by a $10''$ sheet.

7· Draw a tangent to a circle of $2''$ radius, (a) parallel to, (b) perpendicular to, a given straight line, $4''$ away from the centre.

8. Describe a circle to touch two given lines **AB, AC**, at $45°$, through **C**, at $3''$ from **A**.

9. Draw a chord through a given point in a circle which will be bisected at the point.

10. Draw neat figures and verify the following theorems with the help of the instruments :

(1) Angles in the same segment are all equal. Those on a semicircle are right angles.

(2) Two tangents to a circle from a point are equal.

(3) If a chord be drawn through the point of contact of a tangent, the angle between them is equal to the angles in the alternate segments.

(4) The rectangle contained by the segments of two intersecting chords are equal.

(5) Same relations as (4), hold even when the chords meet outside the circle. In addition, the rectangles are equal to the square on the tangent from the point of intersection.

(6) The bisector of the vertical angle of a triangle divides the base in the ratio of the sides.

————:o:————

15 *Areas.* In practice, it is often required to find the areas of quadrilaterals and to divide them into a number of equal parts. The problems in arts. 12 and 13 will be useful in finding equivalent areas and making partition of fields, etc. When the area is bound wholly or partly by curved lines, the following method is used.

Let us find the area of irregular figures as shown.

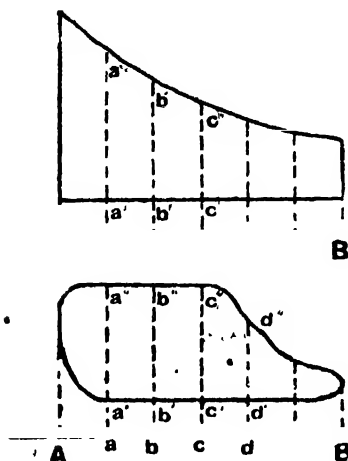Draw a horizontal line **AB** (Fig. 43) below the figure and vertical lines to touch the extremities, of the figures,

Fig 43

cutting **AB** in **A** and **B**. Divide **AB** into a number of equal parts at a, b, etc. Draw vertical lines at every division, to cut the perimeter of the figure at a′ a″, b′ b″ &c. Measure these heights, which are called ordinates, and odd. Divide the sum by the number of the ordinates, to obtain the mean height. This multiplied by the length **AB**, gives the area.

The lower figure shown, occurs in steam engineering.

Simpson Rule to find areas is as follows :

Divide the base into an even number of equal parts, each equal to a say, and measure all the corresponding ordinates.

Then area of curve is equal to

$$\tfrac{1}{3}a \left\{ \begin{array}{l} \text{twice sum of} \\ \text{even ordinates} \end{array} + \begin{array}{l} \text{four times sum} \\ \text{of odd ordinates} \end{array} + \begin{array}{l} \text{sum of first and} \\ \text{last ordinates.} \end{array} \right\}$$

Examples and exercises will be found in section 5

SECTION 4.

Right-angled triangles. Rectangles. Squares. Offsets. Other triangles. Quadrilaterals. Circles.

16. (*a*) The square on the hypotenuse is equal to the sum of the squares on the two other sides of the right angled triangle.

If **H** be the hypotenuse, **A**, the altitude and **B**, the base,

$$H^2 = A^2 + B^2.$$

(*b*) When any two of the three sides of a right-angled triangle are given, the third is easily found.

We have, $A^2 = H^2 - B^2$

$$B^2 = H^2 - A^2$$

B

Fig. 44

Hence

$$H = \sqrt{A^2 + B^2}$$

$$A = \sqrt{H^2 - B^2} = \sqrt{(H+B)(H-B)}$$

$$B = \sqrt{H^2 - A^2} = \sqrt{(H+A)(H-A)}$$

(*c*) The hypotenuse may be found out when the sum of the hypotenuse and one side and the remaining side be given.

Since $\quad H^2 = A^2 + B^2$

$$2H^2 + 2HA = A^2 + B^2 + H^2 + 2HA$$

$$2H(H+A) = B^2 + (H+A)^2$$

or $\quad H = \frac{1}{2} \left\{ \frac{B^2}{H+A} + (H+A) \right\}$

(*d*) The hypotenuse may be found when the difference between the hypotenuse and one side and the remaining side be given.

We have $H^2 = A^2 + B^2$;

$$2H^2 - 2HA = A^2 + B^2 + H^2 - 2HA$$

$$2H(H - A) = B^2 + (H - A)^2$$

$$\text{or } H = \tfrac{1}{2} \left\{ \frac{B^2}{H - A} + (H - A) \right\}$$

Hence the process in (c) & (d) is to divide the square of the given side by the given sum or difference and to the quotient add the same sum or difference ; half the last result is the hypotenuse required.

Examples. 1. The sides of a right-angled triangle are 6 feet and 8 feet ; find the hypotenuse.

$$H = \sqrt{A^2 + B^2}.$$

$$\therefore \quad H = \sqrt{6^2 + 8^2} = \sqrt{100} = 10 \text{ feet.}$$

2. The hypotenuse is 10 feet and one side is 8 feet ; find the remaining side of the right-angled triangle.

$$A = \sqrt{H^2 - B^2}.$$

$$\therefore \quad A = \sqrt{10^2 - 8^2} = \sqrt{100 - 64} = \sqrt{36} = 6 \text{ feet.}$$

3. The hypotenuse is 9 feet and one side is 6 feet ; find the other side.

$$A = \sqrt{H^2 - B^2}.$$

$$= \sqrt{(H + B)(H - B)}.$$

$$\therefore \quad A = \sqrt{(9 + 6)(9 - 6)} = \sqrt{15 \times 3} = \sqrt{45}$$

$$= \sqrt{9 \times 5} \text{ ft.} = 3\sqrt{5} \text{ ft.} = 3 \times 2 \cdot 24 \text{ ft.} = 6 \cdot 72 \text{ feet.}$$

4. The sum of the hypotenuse and one side is 18 feet and the remaining side is 6 feet ; find the hypotenuse.

$$H = \tfrac{1}{2}\left(\frac{6^2}{18} + 18\right) = \tfrac{1}{2}\left(\frac{36}{18} + 18\right) = \tfrac{1}{2}(2 + 18)$$
$$= \tfrac{1}{2} \times 20 = 10 \text{ feet.}$$

5. The difference between the hypotenuse and one side is 14 feet ; the remaining side is 28 feet ; find the hypotenuse.

$$H = \tfrac{1}{2}\left(\frac{28^2}{14} + 14\right) = \tfrac{1}{2}(56 + 14) = 35 \text{ feet.}$$

6. In a triangle **ABC** of which the base **BC** is 30′ a perpendicular **AD** is drawn from **A** on **BC** ; if **AD** is 8′ and **AB**, 10 feet, find **AC**.

Fig. 45

$$BD = \sqrt{10^2 - 8^2} = \sqrt{100 - 64} = \sqrt{36} = 6'.$$
$$\therefore CD = 30 - 6 = 24' ;$$
$$AC = \sqrt{8^2 + 24^2} = \sqrt{64 + 576} = \sqrt{640} = 25{\cdot}29 \text{ feet.}$$

7. A man travels **AB**, 25 miles due North from **A** then **BC**, 10 miles due East and then finally **CD**, 25 miles due North ; what is his distance **AD** from the starting point ?
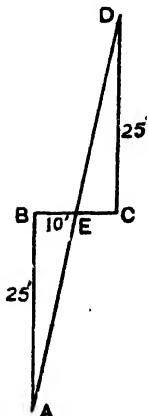
Let **BC** cut **AD** at **E**.

$$AD = AE + ED = 2AE = 2\sqrt{25^2 + 5^2}$$
$$= 2\sqrt{625 + 25} = 2\sqrt{650} = 50{\cdot}98 \text{ miles.}$$

Fig. 46

8. The cost of fencing a rectangular field, at 6 pies per cubit, is Rs. 6/4/- ; if the field be 75 cubits long, find its width. Rs. 6/4 = 100 annas.

Perimeter of the rectangle = 100 × 2 = 200 cubits.

Twice the breadth = 200 − 150 = 50 ∴ breadth = 25 cubits.

9.   A square enclosure has its diagonal 50′ ; what time will be required to walk around it at the rate of one mile an hour.

Diagonal is 50′ ;   ∴ side = 50÷√2.

$$4 \text{ sides} = 50÷√2 × 4 = 50 × 2 × √2 \text{ ft.}$$

1 mile per hour = 1760 × 3÷60 = 88 ft per minute.

∴ the required time

$$= 50 × 2 × √2÷88 = 25√2÷22 = 1·61 \text{ minutes.}$$

10.   The length of a rectangle is 20 feet ; its diagonal is 25 ft. ; find the cost of fencing round it at the rate of one anna per foot.

Length = 20 ft., diagonal = 25 feet ;   ∴ breadth = 15′ ;

∴ the perimeter = 70 feet :

cost is 70 annas = Rs. 4/6/- annas.

11.   The diagonal of a rectangle is 75 ft. ; one side is 45 ft. ; what will be the length of the diagonal of a square of equal perimeter ?

Diagonal = 75 ft., one side = 45′; ∴ the other side = 60′ ;

∴ the perimeter = 210 ft. ;

side of a square of equal perimeter = 210÷4 = 105÷2 ft. ;

∴ the diagonal of the square = 105÷2 × √2

$$= 105÷1·414/2 = 74·235 \text{ feet.}$$

EXERCISE 15.

1.   Find the hypotenuse when the sides of a right-angled triangle are given as follows :

(a)	3 and 4 units of length.	(d)	25 yards and 33 yds. 1 foot.
(b)	30 and 72 ,,	(e)	1 foot 1 inch and 7 feet.
(c)	21 and 7 ,,	(f)	1 Bigha 5 cubits and 1 Bigha 52 cubits.

2. Find the side of a right-angled triangle when the hypotenuse and another side are given as follows .

  (a) 25 and 24 units of length
  (b) 6 5 and 5·4   ''
  (c) 74 and 70   ''
  (d) 4 ft. 5 ins and 3 ft 9 ins.
  (e) 13 yd. 2 ft. and 13 yd 1 ft.
  (f) 2 bighas 2 cubits and 1 bigha 56 cubits.

3. A man on one side of a brook finds that he can just rest a ladder 20 ft. long against the branch of a tree vertically over the other bank ; the branch is 12 ft. above the ground ; how wide is the brook ?

4. The span of a roof is 21 ft., and the rise 7 ft ; find the sloped length of each side.

5. One side of a right-angled triangle is 588 ft. ; the sum of the hypotenuse and the other side is 882 ft Find the hypotenuse and the other side.

6. One side of a right-angled triangle is 3925 ft. ; the difference between the hypotenuse and the other side is 625 ft. ; find the hypotenuse and the other side.

7. A ladder, 25 ft. long, is placed against a wall with its foot, 7', from the wall ; how far should the foot be drawn out so that the top of the ladder may come down by half the distance, that the foot is drawn out.

8. The hypotenuse of a right-angled triangle is 123 ft. and one side is 9 yds. ; find the other side.

9 A ladder 5 ft long, is standing against a wall with its foot 3 ft from the wall ; how far should the foot be drawn out so that top of the ladder is 3 ft. from the ground ?

10. Find the cost of fencing the sides of a rectangular field, of which the diagonal is 162 ft. and a side 136 ft., at 4 as. per foot.

11. How many posts are required, if placed, 2 ft. apart, along the diagonal of a rectangular yard of 6 and 8 ft. sides ?

12. It costs Rs. 204 to fence a right-angled triangular field and Rs. 51 only for fencing one side. If it costs Rs 1 per foot, find the length of the other side.

13   If the difference in times taken to walk along the hypotenuse
and one side of a path-way in the form of a right-angled triangle
be 14 minutes and it takes 28 minutes to walk at the same rate
along the other side, find the length of hypotenuse, if the rate be
100 yards per minute.

14.   Find the diagonal of a square, whose side is $24\sqrt{2}$ inches.

15.   If Calcutta be 320 miles due South of Darjeeling and
Chittagong, 224 miles due East of Calcutta, how far is Chittagong
from Darjeeling ?

16   What is the biggest straight line that can be drawn on a piece
of paper, 9 ins. by 16 ins ?

17   One side of a right-angled triangle is 20 feet   the difference
between its hypotenuse and the other side is 13' : find the hypotenuse.

18   One side of a right-angled triangle is 12 feet and the sum of
its hypotenuse and the other side is 45 feet : find the hypotenuse.

17. (a) In an isosceles right-angled triangle, the
hypotenuse is $\sqrt{2}$ times of either side. This follows
from 16 (a), when $A = B$,

$$H^2 = A^2 + B^2 = 2A^2 \text{ or } 2B^2 ;$$

$$\text{or } H = \sqrt{2}A \text{ or } \sqrt{2}B.$$

Or, each side is hypotenuse divided by $\sqrt{2}$.

(b) If two sides of a right-angled triangle be

Fig. 47

given, the length of the
perpendicular from the right
angle on the hypotenuse can
be found out, by dividing the
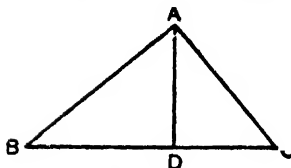product of the two sides by
the hypotenuse.

If **AD** is the perpendicular to hypotenuse **BC**, **AB**
and **AC** are two sides, then **AD : AC : : AB : BC** ;

therefore $AD = \dfrac{AC \times AB}{BC}$

The perpendicular is called an off-set.

(c)   If in the above, the segments of the hypotenuse **BC** be given, the perpendicular **AD** is found by extracting the square root of the product of the segments. Since **BD : AD : : AD : DC, AD' = BD· DC.**

Examples. 1.· The sides of a right-angled isosceles triangle are each $3\sqrt{2}$ inches.   What is the length of the hypotenuse ?

Required length = $\sqrt{2} \times 3\sqrt{2} = 6$ inches.

2.   The two sides of a right-angled triangle are 3 and 4 inches.   Find the length of the perpendicular from the right angle to the hypotenuse.

Hypotenuse is $\sqrt{3^2 + 4^2} = 5$

Required perpendicular is $\dfrac{3 \times 4}{5} = \dfrac{12}{5} = \dfrac{24}{10} = 2\cdot4$ inches.

3   If the perpendicular from a right angle to the hypotenuse of a right-angled triangle divide the hypotenuse of 13 inches into two parts of 9 and 4 inches, find the length of the perpendicular.

Required perpendicular = $\sqrt{4 \times 9} = 6$ inches.

4.   **ABCD** is a rectangle ; the diagonal **AC** is divided into two parts, 8 ins. and 18 ins. by a perpendicular drawn on it from **B**.   Find the sides of the rectangle.

DE = $\sqrt{18 \times 8} = \sqrt{144} = 12$
DC = $\sqrt{144 + 64} = \sqrt{208} = 14\cdot42$
AD = $\sqrt{324 + 144} = \sqrt{468} = 21\cdot63$ ins.

5.   The length of a rectangle **ABCD** is 30' and the breadth 8 ft.   The side **AD** is divided into three equal parts at **E** and **F** and straight lines, parallel to **AB** are

drawn through them, cutting the side **BC** at **G** and **H**.
EG and **FH** are bisected at **K** and **L**. Find the lengh
of the line **DLKB**.
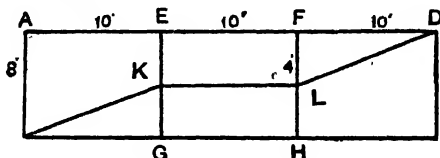
Fig 48.

$$DL = \sqrt{10^2 + 4^2} = \sqrt{116} = 10\!\cdot\!77$$
$$BK = \sqrt{10^2 + 4^2} = \sqrt{116} = 10\!\cdot\!77$$
$$KL \qquad\qquad\qquad = 10$$
$$\qquad\qquad\qquad\qquad 31\!\cdot\!54 \text{ feet.}$$

EXERCISE 16.

1   Find the value of $\sqrt{2}$ to 5 places of decimals.

2.   Find the diagonal of a square of 7 inch sides to 3 places of decimals

3.   If the diagonal of a square be 10 inches, find its sides.

4.   If it takes 15 minutes to walk along the diagonal of a square field, how long will it take to go round it ?

5.   The sides of a rectangle are 6 and 8 inches. Find the lengths of perpendiculars from the angular points to the diagonals.

6.   The diameter of a circle is 12 inches. The length of a perpendicular on the diameter up to the circumference, measures 4 inches. Find the point on the diameter on which the perpendicular stands.

7.   A rectangle is drawn inside a circle of 5 feet diameter. If the sides be 3 and 4 ft. find the lengths of perpendicular from the angular points to the diameter.

8.   In the above example, find the points on the diameter on which the perpendicular stand.

9.   Two stations are 35 miles apart. A third station is 28 and 21 miles respectively from the two. If one goes straight from the first

to the second how near would he approach the third station during the journey ?

10. Find the cost of fencing round a square field, if the cost of fencing along the diagonal be Rs. 40/.

11. The perimeter and the hypotenuse of a right-angled triangle measure 24 and 10 inches respectively. Find the length of the perpendicular from the right angle to the hypotenuse

12. If the perimeter of an isosceles right-angled triangle be 18 inches, find the hypotenuse.

———— .0. ————

18. (a) The altitude of an equilateral triangle may be found out, if the side be given.

$$\text{Altitude} = \sqrt{(\text{side})^2 - \left(\frac{\text{side}}{2}\right)^2}$$

$$= \tfrac{1}{2}\sqrt{3} \text{ of a side.}$$

Also side $= \dfrac{2}{\sqrt{3}}$ altitude.

The side of the equilateral triangle is to be multiplied by $\tfrac{1}{2}\sqrt{3}$ or by 0·866 ; the product is the altitude required.

Fig. 49

(b) If the base b, and a side, a, of an isosceles triangle be given, the altitude h, of the triangle may be found out as follows :

Subtract the square of half the base from the square of any of the equal sides ; the square root of the difference is the altitude required.
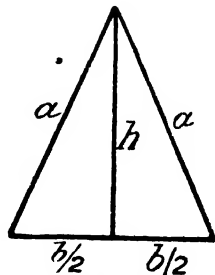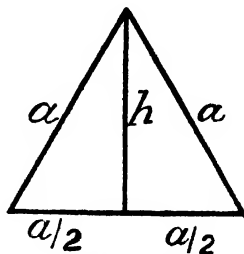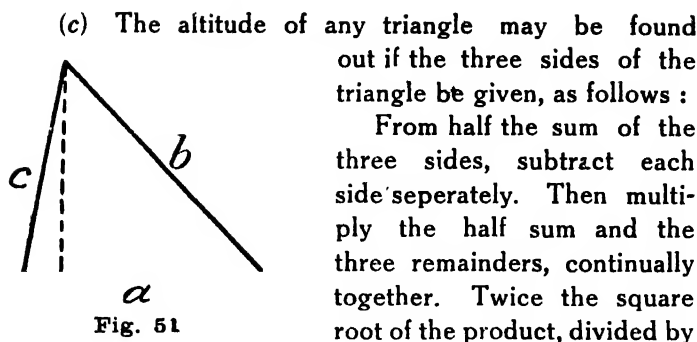

Fig. 50

(c) The altitude of any triangle may be found out if the three sides of the triangle be given, as follows :

Fig. 51

From half the sum of the three sides, subtract each side seperately. Then multiply the half sum and the three remainders, continually together. Twice the square root of the product, divided by any of the side considered as the base, is the altitude required.

Let $2s = a + b + c$, sum of the 3 sides.

Then $\dfrac{2\sqrt{s(s - a)\,(s - b)\,(s - c)}}{a}$ is the length of the perpendicular on a : and so on.

Examples. 1. A side of an equilateral triangle is 5 feet, find the height of the triangle.

Half side = $\frac{5}{2}$ ft.

Altitude $= \sqrt{5^2 - (\frac{5}{2})^2} = \sqrt{(25 - \frac{25}{4})} = \sqrt{18\frac{3}{4}}$

$= 4\text{·}33$ feet.

or, altitude $= \frac{1}{2} \times \sqrt{3}$ side, i.e., $5 \times 0\text{·}866$, or $4\text{·}33$ feet.

2. An isosceles triangle has its base 10 feet long and each of the equal sides, $12' - 6''$; what will be the diagonal of a square whose side is equal to the altitude of the isosceles triangle ?

Side of the isosceles triangle $= 12' - 6''$; half base, 5 ft ;

$\therefore$ the altitude $= \sqrt{(12\frac{1}{2})^2 - 5^2} = \sqrt{625/4 - 25}$

$$= \sqrt{\dfrac{625 - 100}{4}} = \sqrt{\dfrac{525}{4}}$$

∴ the diagonal of the square $= \sqrt{525/4} \times \sqrt{2}$
$$= \sqrt{525/2} = \sqrt{262\cdot 5} = 16\cdot 2 \text{ ft.}$$

3. Three sides of a triangle are 5, 7, 8 feet ; find the altitude on the side, 8 feet long.

$\frac{1}{2} (5+7+8) = 10 ;$
$10 - 5 = 5$
$10 - 7 = 3$
$10 - 8 = 2$

The altitude $= \dfrac{2\sqrt{(10 \times 5 \times 3 \times 2)}}{8}$

$= \dfrac{2\sqrt{5 \times 5 \times 2 \times 2 \times 3}}{8} = \dfrac{2 \times 5 \times 2\sqrt{3}}{8}$

$= \dfrac{20\sqrt{3}}{8} = \dfrac{20 \times 1\cdot 73205}{8} = \dfrac{5 \times 1\cdot 73205}{}$

$= 4\cdot 33 \text{ feet.}$

4. In an equilateral triangle, the perpendicular drawn from the vertex to the base is $4\sqrt{3}$ ft ; find its side.

Perp. $= 4\sqrt{3}$; ∴ half base is 4 ft. ;

∴ the side is $4 \times 2 = 8$ ft.

5. The diagonal of a square is $7\cdot 07$ inches ; what will be the height of an equilateral triangle, whose side is equal to the side of the square ?

Diagonal $= 7\cdot 07$; side $= 7\cdot 07/1\cdot 414 = 5$ inches.

∴ the side of the equilateral triangle $= 5''$;

∴ height $= \frac{5}{2}\sqrt{3} = 4\cdot 33$ inches, nearly.

6. An equilateral triangle and a square are drawn on the same base, which is 20 ft. long. Find the difference between the diagonal of the square and the height of the triangle.

Diagonal of square $= 20\sqrt{2} = 28\cdot 28''$
Height of equilateral triangle $= 10\sqrt{3} = 17\cdot 32''.$

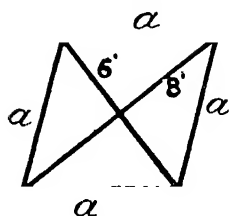Difference $28\cdot28'' - 17\cdot32'' = 10\cdot96$ inches.

Fig. 52

7.   Half diagonals of a rhombus are 6 ft. and 8 ft.   Find its side, a.
Side, $a = \sqrt{8^2 + 6^2} = 10$ feet.

8.   One side of a rhombus is 10 yards and one of its diagonals is 12 yards.   Find the other diagonal.

Half of the other diagonal $= \sqrt{10^2 - (\frac{12}{2})^2} = 8$ yards.

Hence the second diagonal is 16 yards.

If the perimeter were given, side $= \frac{1}{4}$ perimeter.

9.   One angle of a rhombus is 60 degrees; if its side be 8 feet, find its diagonals.

Side $= 8$ ft.   Therefore, the diagonal opposite to 60 deg. angle, is 8 ft. and another diagonal is $2\sqrt{8^2 - 4^2}$

$\qquad = 2\sqrt{12 \times 4} = 2 \times 4\sqrt{3} = 13\cdot856$ feet.
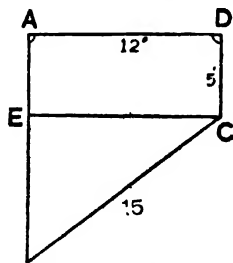
10.   **ABCD** is a quadrilateral; **AD** $= 12$ ft.   **DC** $= 5$ ft.;

Fig. 53

BC $= 15$ ft.;   and   the $\angle$**ADC** $= \angle$**DAB** $= 90$ degrees; find **AB**.

Draw **CE** perp. to **AB**. Then $\mathbf{EB} = \sqrt{15^2 - 12^2} = 9$.
$\mathbf{AB} = \mathbf{AE} + \mathbf{EB} = 5 + 9 = 14$ feet.

11. **ABCD** is a quadrilateral; $\angle \mathbf{BDC} = \angle \mathbf{DCB}$, each equal to 60 degrees. **AD** is 40 ft. $\angle \mathbf{BAD}$ is 90 degrees and **BD**, 50 ft. ; find **AB**.

$\angle \mathbf{BDC} = 60$ deg. $\angle \mathbf{DCB} = 60$ deg. $\therefore \angle \mathbf{CBD} = 60$ deg.

Therefore the triangle **CBD** is equilateral.

The triangle **BAD** is right angled, therefore

$$\mathbf{AB} = \sqrt{\mathbf{BD}^2 - \mathbf{AD}^2} = 30 \text{ ft.}$$

Fig. 54

12. In a quadrilateral, the angles at **B** and **D** are right angles ; **AB** is 30 ft. ; **BC**, 40 ft. **DC**, 45 ft. ; find **AD**. **AC** = 50′.
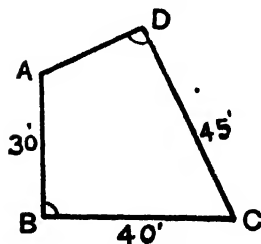
$$\therefore \ \mathbf{AD} = \sqrt{(50 + 45)\ (50 - 45)}$$
$$= \sqrt{95 \times 5} = \sqrt{19 \times 5 \times 5} = 5\sqrt{19}$$
$$= 5 \times 4\cdot36 = 21\cdot8 \text{ feet.}$$

Fig. 55

13. **ABCD** is a quadrilateral ; its diagonal **AC** is 20 ft. The perpendicular on **AC** from **B** is 8 ft. and divides **AC** into two parts, 4 ft. and 16 ft ; also, the perpendicular on **AC** from **D** is 6 ft. and divides **AC** into two parts, 8 ft. and 12 ft. ; find the four sides of the quadrilateral.

$\mathbf{BC} = \sqrt{64 + 16} = \sqrt{80} = 4\sqrt{5} = 4 \times 2\cdot236 = 8\cdot944$.

AB is similarly $= \sqrt{256 + 64} = \sqrt{320} = 8\sqrt{5} = 17\cdot888$.

$\mathbf{AD} = \sqrt{64 + 36} = \sqrt{100} = 10$:

and $\mathbf{DC} = \sqrt{144 + 36} = \sqrt{180} = \sqrt{36 \times 5} = 6\sqrt{5} = 13\cdot416$.
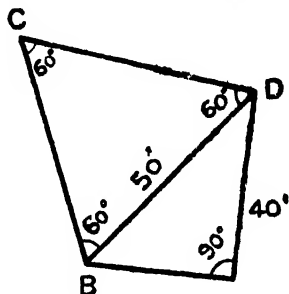
14. The two roofs of a *deshi* hut meets at the top at an angle of 90 degrees ; width of the roofs 16′ and 12′ ; the corner posts are 8′ high. Find the height of the ridge.
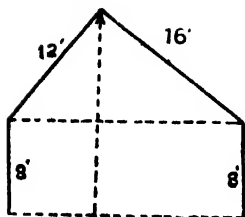

Fig. 56

$$\text{Span}^2 = \sqrt{12^2 + 16^2} \therefore \text{ span} = 20.$$

Height of ridge above post

$$= \frac{16 \times 12}{20} = 9\tfrac{3}{5} \text{ ft.,}$$

since $\dfrac{12 \times 16}{2} = \dfrac{20 \times \text{height}}{2} = $ area of triangle.

$\therefore$ the height of top from ground is $9\tfrac{3}{5} + 8' = 17\tfrac{3}{5}$ feet.

˙ EXERCISE 17.

1. Calculate the value of $\sqrt{3}$ to 3 places of decimals.

2. If the perpendicular from the vertex to the base of an equilateral triangle be $4\sqrt{3}$ inches, find the side.

3. Find the altitude of an equilateral triangle of $3\sqrt{3}$ inch sides.

4. The altitude of an equilateral triangle is $\sqrt{3} \times 5$ inches. Find the side.

5. The side of an equilateral triangle is $200\sqrt{3}$ ft. What is the height ?

6. Calculate the altitude of an equilateral triangle, if the sides are 30 ft. each, to the nearest inch.

7. Find the cost to the nearest pie, of fencing at the rate of annas ./6/- per foot, a triangular field of equal sides, if the perpendicular distance from an angular point to the opposite side be 40 ft.

8. Each of the equal sides of an isosceles triangle is 10 ft. and the base is 12 ft. Find the altitude of the triangle.

9. The side of a square is 6 yds. Find the radius of the circle described round the square.

10. The radius of a circle is 1 ft. Find the area of a square inscribed in the circle.

11. If the sides of any triangle be 3, 4 and 5 inches, find the lengths of the perpendiculars to the sides from opposite angular points.

12. The sides of a triangle are 25, 39 and 56 ft. respectively ; find the perpendiculars from the opposite angles on the side of 56 ft.

13. The three sides **AB**, **AC** and **BC** of the triangle **ABC** are 68, 75 and 77 ft. respectively ; find the length of the perpendicular from **A** on **BC**.

14. The sides of a triangle are 13, 14 and 15 ft. ; find the perpendicular from the opposite angle on the side of 14 ft.

15. A house 42 ft. wide, has a roof with unequal slopes, the lengths of which are 26 and 40 ft : find the height of the ridge above the eaves.

16. The sides of a triangle are in the ratio of 13, 14, 15 and the perimeter is 84 yds. : find the perpendiculars from the angular points upon the sides.

17. The sides of a triangle are 25, 101, 114. Find the two parts into which the longest side is divided by the perpendicular from the opposite angle.

18. The diagonals of a rhombus are 72 and 96 ft. Find the length of its sides.

19. The semi-diagonals of a rhombus are 8 and 16 inches respectively. Find the length of its sides.

20. The diagonals of a rhombus are respectively (1) 40 and 60 yards, (2) 88 and 234 ft. ; find the perimeter and height.

21. The sides of a rhombus is 36 ft. and one of its diagonals is 18 ft. Find the other diagonal.

22. The side of a rhombus is 20, and its longer diagonal is 34.64 ; find the other diagonal.

23. The diagonals of a rhombus are (1) 60 ft. and 45 ft., (2) 4 ft. and 1 ft. 2 inches, (3) 80 ft. and 60 ft. Find the side and the height.

24. In the quadrilateral **ABCD**,

(1) **AD**=16, **DC**=8; **BC** = 12 **LADC**=∠**DAB**=90°; find **AB**.

(2) **LBDC**−**LDCB**=60°;   **LBAD**=90°;   **AD**=60 ft.,
BD=80 ft.; find **AB**.

(3) ∠**B**=∠**D**=90°;   **AB**=40 ft.; **BC**=50 ft.,   **DC**=60 ft.;
find **AD**.

(*d*) If **ABC** and **DEF** are two given similar triangles,
AC : **BC** : : **DF** : **EF**.
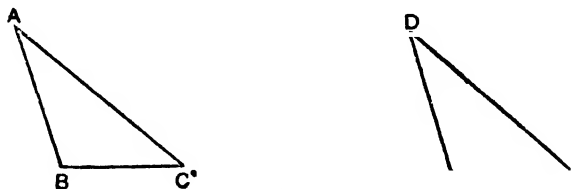
Fig. 57

Hence **DF** is equal to $\dfrac{CA \times EF}{BC}$.

Let **BC** be 5 feet, **CA**, 8 feet and **EF**, 10 feet ; then,
DF will be

$$\frac{8 \times 10}{5} = 16 \text{ feet.}$$

(*e*) Similar rectilinear figures having their angles
equal, each to each, have their sides about the equal
angles proportional. They may also be divided into
the same number of similar triangles.

If two straight lines of one rectilinear figure and a
straight line corresponding to one of them of a similar
figure be given, the straight line corresponding to
the other is found out in the same way, as in the case
of similar triangles.

(*f*) Two maps, drawn on different scales of the same place, are similar ; a line drawn on a map prepared on the scale of 32″ to a mile, will be twice as long as the corresponding line drawn on the other map prepared on the scale of 16″ inches to a mile.

Examples. 1. The height of a post is 10 feet and the length of its shadow is 15 feet. The length of shadow of another post is found to be 42 feet at the same time. Find the height of the post.

Height 10′ : shadow 15′ : : reqd. height : its shadow 42′

$$\therefore \quad \text{the required height} = \frac{10 \times 42}{15} = 28 \text{ feet.}$$

2. There are two poles 120′ and 40′ long at a distance of 60′. Strings are attached to the top of each and foot of the other. Find the height of the point above the ground at which the two strings cross.

$\dfrac{AB}{BC} = \dfrac{EF}{FC}$ $\therefore$ $EF = \dfrac{AB \times FC}{BC}$

$\dfrac{DC}{BC} = \dfrac{EF}{BF}$ $\therefore$ $EF = \dfrac{DC \times BF}{BC}$

$\therefore$ $\dfrac{AB \times FC}{BC} = \dfrac{DC \times BF}{BC}$

$\therefore$ $AB \times FC = DC \times BF$

$\therefore$ $\dfrac{AB}{DC} = \dfrac{BF}{FC}.$ But AB is 3 DC

$\therefore$ $BF = 3 FC$: $\therefore$ BC is 4 FC ;

$\therefore$ $FC = 15$ and $BF = 45$ ft.
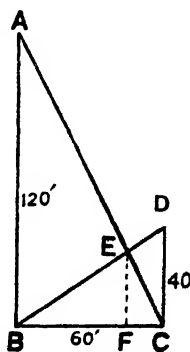
Now $\dfrac{BF}{FE} = \dfrac{BC}{CD}$ $\therefore$ $\dfrac{45}{EF} = \dfrac{60}{40}$; i.e., $EF = \dfrac{45 \times 40}{60} = 30$ ft.

Fig. 58

3. The shadow of a man 6 ft. high, is found to be 4'-6"; what will be the length of the shadow of a post 15 feet high at that time ?

$$6 : 4\tfrac{1}{2} :: 15 : x \; ; \; \text{or } x = \frac{15 \times 9}{2 \times 6} = 45/4 = 11\tfrac{1}{4} \text{ feet.}$$
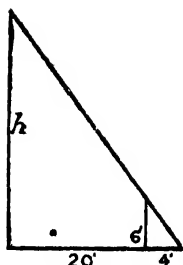
Fig. 59

4. A man 6' in height standing 20' from a lamp post observes that his shadow cast by the light is 4 feet long ; find the height of the post.

Height : 24 :: 6 : 4

∴ height = 24 × 6/4 = 36 feet.

5. At a distance of 300 feet from a building, the shadow of a vertical stick 4 ft. high, cast by light from its top, is observed to be 16 ft. long ; what is the height of the building ?

The shadow of 4 ft. stick is 16 ft. ; 16' : 4' = 316' : x.

$$\text{or } x = \frac{4 \times 316}{16} = 79 \text{ feet.}$$

6. The length and breadth of a building are 100 and 80 feet respectively ; the length of its plan is 5 ins. ; what would be the breadth ?

100' is represented by 5″ ; ∴ 20' by 1″ or, 80' by 4″.

7. What will be the length of a line 13200 ft. long in a map, which is drawn on a scale, 20 chains = 1 inch ?

20 ch = 20 × 66 = 1320 ft. is represented by 1 inch.

∴ 13200 ft. is represented by 10 inches.

8. The diameter of a circle is 3 feet ; find the side of the inscribed equilateral triangle.

Diameter = 36″ ; radius is 18″

∴ half the side of the ins-
cribed equilateral triangle
= 9√3.

∴ side = 18√3 = 18 × 1·732
= 31·18 inches.
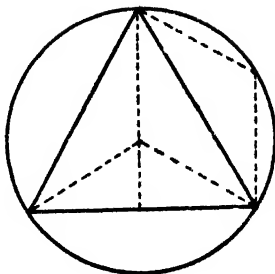
Fig. 60

9. The diameter of a circle is 4 feet ; find the side of the inscribed regular hexagon.

Diameter = 4 ft. = 48″ ; ∴ radius = 24″.

∴ side of the hexagon is 24″ inches.

10. The radius of a circle is 10 ft. ; find the side of the inscribed dodecagon.

Radius – perp. to side of inscribed hexagon, from centre = 10 – 5√2 [·§ 18 (a)]

= 5 (2 – √3) = 5 × 0·268 = h (say).

Since side : h = diameter : side, [fig. 61,§ 19 (a)]

(side)² = diameter × h = 20 × 5 × 0·268,

or side = √(100 × 0·268) = √26·8 = 5·18 feet.

11. The diameter of a circle is 10 inches ; find the side of the circumscribed equilateral triangle.

Radius = 5 inches.

∴ the height of the circumscribed triangle is 15 inches, the centre being the point of intersection of the medians.

∴ half base of the triangle = 15/√3

∴ base = 2 × 15/√3 = 30/√3 = 10√3 = 17·32 inches.

12. The side of a regular octagon is 6″; find the radius of the inscribed circle

Radius of the inscribed circle $= \frac{1}{2}$ diameter

$$= \tfrac{1}{2}\ (6/\sqrt{2}+6+6/\sqrt{2}) = \tfrac{6}{2}\left(\frac{2}{\sqrt{2}} + 1 \right)$$

$$= 3\ (\sqrt{2}+1) = 3\ (1\cdot414+1) = 3 \times 2\cdot414 = 7\cdot242.$$

$$\left|\ \text{approx., } = \text{side} \times 1\cdot2\ \right|$$

EXERCISE 18.

1. The section of a canal is 32 ft. wide at the top, 14 ft. wide at the bottom and 8 ft. deep.   If the surface of the water be 26 ft wide, what is its depth ?

2. The parallel sides of a trapezoid are respectively 8 ft. and 14 ft , two straight lines are drawn across the figure parallel to these, at the same distance from the parallel sides, as between them Find the lengths of the straight lines.

3. The parallel sides of a trapezoid are respectively 16 and 20 ft and the perpendicular distance between them is 5 ft. ; the other two sides are produced to meet. Find the perpendicular distance of the point of intersection from the longer of the two parallel sides.

4. If the length of the shadow of a man $5\frac{1}{2}$ ft. high, be 13 ft , what is the length of the shadow of a building 40 ft. high ?

5. If a kite is flying at a height of 300 ft. with a line of 1000 ft., what is the height of the middle point of the line ?

6. A room, 24 ft. by 16 ft. is shown in plan of which the breadth is 4 inches. What is the length of the plan of the room ?

7. If the distance from London to Calcutta, viz , 4000 miles, is shown on a map by 16 inches, what length will be shown in the map for the distance between Calcutta and Bombay, of 1200 miles ?

8. If the diameter of a circle be 6 inches, what is the side of an inscribed equilateral triangle ?   What is it, of an inscribed regular hexagon ?

9. The radius of a circle is 4 inches.   Find the side of the circumscribed equilateral triangle.

10 If the sides of a regular octagon be 4 inches, find the radius of the inscribed circle.

19. Circles are all similar figures.

(a) **AB** be the chord of a circle of which **C** is the centre. **CD** is drawn perpendicular to **AB**, and produced both ways to meet the circumference at **E** and **F**.

Join **AE**, **EB**·and **AF**.

Since **FCDE** is perpendicular to **AB**, **AB** is bisected at **D**, i. e., **AD** = **DB**.

Hence **AE** = **EB**.

AE and **EB** are *chords of half the arc* **AEB**. **DE** is the *rise of the arc* **AEB**.

Since the triangles **AFE**, **DFA** and **ADE** are similar :
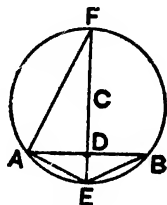
Fig. 61

(1) $\dfrac{ED}{EA} = \dfrac{AE}{EF}$ or, $AE^2 = ED \times EF$;

Hence $ED = \dfrac{EA^2}{EF}$; $EF = \dfrac{EA^2}{ED}$.

That is :

Rise of arc, $ED = \dfrac{EA^2}{EF} = \dfrac{\text{(chord of half the arc)}^2}{\text{diameter of circle}}$

Diameter, $EF = \dfrac{EA^2}{ED} = \dfrac{\text{(chord of half ·the arc)}^2}{\text{rise of arc}}$

Chord of half the arc, $EA = \sqrt{EF \times ED}$

$\qquad\qquad = \sqrt{\text{diameter of circle} \times \text{rise of arc}}$.

Thus, if any two of the 3 quantities be given, the third is found.

Inscribed in a circle of radius **R**,

rise of arc. (i) for an equilat. triangle = $\dfrac{R}{2}$ (fig. 4)

(ii) for a regular hexagon, $= \mathbf{R}$ – perp. to side $\mathbf{F}$

$$= \mathbf{R} - \frac{\sqrt{3}}{2}\mathbf{R} \text{ (§ 18, a)}$$

(iii) for a regular octagon, $= \mathbf{R}$ – radius of inscribed circle

$$= \mathbf{R} - \frac{s}{2}(\sqrt{2}+1),$$

where s is side (ex. 12, page 74)

(iv) for a regular dodecagon, $= \sqrt{s^2 - \dfrac{\mathbf{R}^2}{4}}$;

since in this case, $\mathbf{AD} = \frac{1}{2}\mathbf{AB} = \dfrac{\mathbf{R}}{2}.$

(2) $\dfrac{\mathbf{ED}}{\mathbf{AD}} = \dfrac{\mathbf{AD}}{\mathbf{DF}}$ or $\mathbf{AD}^2 = \mathbf{ED} \times \mathbf{DF}$;

Hence $\mathbf{ED} = \dfrac{\mathbf{AD}^2}{\mathbf{DF}}$; $\mathbf{DF} = \dfrac{\mathbf{AD}^2}{\mathbf{ED}}.$

The chord of circle, $\mathbf{AB} = 2\mathbf{AD} = 2\sqrt{\mathbf{ED} \times \mathbf{DF}}$

$= 2\sqrt{\text{Product of segment of diam. made by the chord}}$

$= 2\sqrt{\mathbf{ED} \times (\mathbf{EF} - \mathbf{ED})}$

$= 2\sqrt{\text{rise of arc} \times (\text{diameter} - \text{rise of arc})}$

$= 2\dfrac{\mathbf{EA}^2}{\mathbf{EF}} \times (\mathbf{EF} - \mathbf{ED})$

$= 2\sqrt{\dfrac{(\text{chord of half arc})^2}{\text{diameter}} \times (\text{diameter} - \text{rise of arc})}$

The diameter of circle, $\mathbf{EF} = \mathbf{ED} + \mathbf{DF}$

$= \mathbf{ED} + \dfrac{\mathbf{AD}^2}{\mathbf{ED}} = \text{rise} + \dfrac{(\text{half chord})}{\text{rise}}$

$\dfrac{\mathbf{EA}^2}{\mathbf{DE}} = \dfrac{\mathbf{EF}^2}{\sqrt{\mathbf{EA}^2 - \mathbf{AD}}} = \dfrac{\mathbf{EA}^2}{\sqrt{\mathbf{EA}^2 - (\frac{1}{2}\mathbf{AB})^2}}$

$$\dfrac{(\text{chord of } \frac{1}{2} \text{ arc})^2}{\sqrt{(\text{chord of } \frac{1}{2} \text{ arc})^2 - (\frac{1}{2}\text{ chord})^2}}$$

(b) The ratio of the length of the circumference of a circle to the length of its diameter is a constant quantity for all circles, but is incommensurable. The ratio $\frac{2\,2}{7}$ is however considered sufficient for practical purposes. A more accurate figure for the ratio is 3·14159265 .....or 3·1416, and is denoted by the Greek letter π.

It is also nearly equal to $\sqrt{10}$ Thus we have, diameter

$$= 2 \times \text{radius} = \frac{\text{circumference}}{\pi}$$

or circumference $= \pi \times \text{diameter}$
$$= 2\pi \text{ radius}.$$

Fig. 62

(c) The angles subtended at the centre of a circle are proportional to the arcs subtended by them.

That is, angle $\dfrac{\textbf{AOB}}{\textbf{BOC}} = \dfrac{\text{arc } \textbf{AB}}{\text{arc } \textbf{BC}}$

The angle at the centre of a circle is 360° and is subtended by the circumference of the circle.

Thus we have, $\dfrac{\text{arc}}{\text{circumference}}$

$$= \frac{\text{angle subtended by the arc}}{360°}.$$

When any two of these 3 quantities are given, the third is easily found out.

(d) The length of an arc is given in terms of the chord of half arc as follows :

Length of arc $= \dfrac{8 \times \text{chord of } \frac{1}{2} \text{ arc} - \text{chord of arc}}{3}$
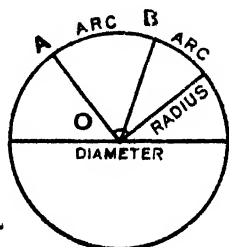
But this rule is not very accurate.

Examples. 1. In a circle, a chord is 12 ins. long and the height of the arc is 3 inches. Find the diameter.

Half chord = 6 ins. ; diameter − rise = $6 \times 6/3 = 12$;

∴ diameter = $12 + 3 = 15$ ins.

2 The height of arc is 7 inches and the chord of half the arc, 35 inches ; find the radius of the circle.

diameter = $\dfrac{35 \times 35}{7} = 175$ ins. ∴ radius = $87\frac{1}{2}$ inches.

3. In a circle of diameter 50 inches, the height of an arc is 5 ins. ; find the chord of half the arc.

Chord of half the arc = $\sqrt{50 \times 5} = \sqrt{250} = 15\,81$ ins.

4. The chord of an arc is 40 inches, and chord of half the arc is 25 inches ; find approximately the length of the arc.

Length = $\dfrac{25 \times 8 - 40}{3} = \dfrac{200 - 40}{3} = \dfrac{160}{3} = 53\,33$ inches.

5. A circular plot of land is 21 ft. radius ; what will be the cost of enclosing it, at anna one per yard ?

Radius is 21 ft. ; circumference = $22/7 \times 42$

$= 132$ ft. = 44 yds.

Cost is 44 annas = Rs. 2-12-0 annas.

6. A man can walk round a circular plot of land in 20 minutes. In what time can he cross it diametrically ?

In, $20 \div \dfrac{22}{7} = 7 \times 10/11 = 70/11 = 6\frac{4}{11}$ minutes.

7. Find the sides of squares inscribed in and circumscribed about a circle of 10 ft. radius.

Side of the circumscribed square

= diameter of the circle = 20 ft.

Side of the inscribed square

= radius × $\sqrt{2} = 10\sqrt{2} = 14\,14$ ft.

8. The diameter of a wheel is 2 feet. How many times will it turn, to go over a circular path of 100 feet radius ?

Diameter of wheel = 2 ft.

∴ circumference $= \frac{22 \times 2}{7} = 44;7$ ft.

Radius of the path 100 ft. ∴ diameter is 200 ft.

∴ circumference $= \frac{22}{7} \times 200$;

∴ number of turns $= \frac{(22 \times 200)}{7} \div \frac{44}{7} = 100$ times.

9. A man takes 4 minutes more to cross a circular plot of land by the circumference than by the diameter, walking at the rate of 2 miles an hour. What is the diameter of the plot ?

2 miles in one hour is equivalent to $2 \times 1760 \times 3$ ft. in 60 minutes, or 176 ft. in 1 minute, or $4 \times 176 = 704$ ft. in 4 minutes. Semicircumference – diameter $= \pi r - 2r = 704$ ft.

diameter $= 2r = \frac{2 \times 704}{\pi - 2} = 1232$ ft.

10. The radius of a circle is 42 inches ; find the length of the arc which subtends angle of 45 degrees at the centre.

Radius is 42 ; that is, diameter is 84 and circumference is $22 \times 84/7$.

∴ arc subtending 45 degrees = 1/8th of the circumference

$$= \frac{22 \times 84 \times 1}{7 \times 8} = 231/7 = 33 \text{ inches.}$$

EXERCISE 19.

1. Find x in the following :—

	Chord.	Rise of arc.	Diameter.	Chord of half arc.
(1)	—	x	14 ins.	2 ins.
(2)	—	x	24 ins.	5 ins.
(3)	—	2 ins.	x	5 ins.
(4)	—	$1\frac{1}{2}$ ins.	x	$3\frac{1}{2}$ ins.
(5)	—	3 ins.	20 ins.	x
(6)	—	1 ins.	15 ins.	x
(7)	x	2 ins.	20 ins.	—
(8)	x	$1\frac{1}{2}$ ins.	15 ins.	—
(9)	x	$1\frac{1}{2}$ ins.	15 ins.	—
(10)	x	2 ins	20 ins.	—
(11)	10 ins.	3 ins.	x	—
(12)	12 ins.	2 ins.	x	—
(13)	16 ins.	—	x	10 ins.
(14)	12 ins.	—	x	8 ins.

2. Find the length of a chord which meets a diameter at right angles and divides it into 2 segments as follows :

(1) 4 and   8 inches .   (2) 2 and   8 inches :
(3) 3 and 12 inches :   (4) 6 and 14 inches :
(5) 5 and 15 inches :   (6) $2\frac{1}{3}$ and $11\frac{1}{2}$ inches. :

3. Find x in the following :—

	Diameter.	Radius.	Circumference, ins.
(1)	—	x	14
(2)	x	—	25
(3)	x	—	28
(4)	x	—	94
(5)	—	x	65
(6)	—	x	86
(7)	—	x	58
(8)	x	—	144

4. Find the length of an arc of a circle of (a) 12 inches circumference, (b) 8 inches radius, which subtends the following angle at the centre—

(1) 60° :   (2) 45° :   (3) 30° :   (4) 75° :
(5) 90° :   (6) 120° :   (7) 36° :   (8) 100° :

5. A string is coiled 24 times on a cylinder, 4 inches in diameter ; what is the length of the string ?

6. A man, by walking diametrically across a circular grass plot, finds that it has taken him 45 seconds less than if he had kept to the path round the outside. If he walks 80 yards a minute, what is the diameter of the grass plot ?

7. The difference between the circumference and diameter of a circle is 60 ft. Find the radius.

8. The times taken by a cyclist going at a steady rate, respectively round the outer and inner edges of a circular track, are as 23 to 22, and the width of the track is 15 ft. Find the diameter of the circle forming the inner edge of the track.

9. A road runs round a circular plot of ground ; the outer circum‑ ference of the road is 44 yards longer than the inner ; find the breadth of the road.

10 The chord of an arc is 5 feet, and the diameter of the circle is 7 feet , find the height of the arc in inches.

11. The chord of an arc is 8 yards and the chord of half the arc is 13 feet. Find the diameter of the circle. •

12. The chord of an arc is 10 feet, and the height of the arc is 2 yards. Find the diameter of the circle and the chord of half the arc.

13. The chord of an arc is 36 feet, and the chord of half the arc is 19½ feet. Find the diameter of the circle.

14. Find the length of an arc of a circle, whose radius is 6 feet, the chord of the arc being 8 feet.

15. The radius of a circle is 7 feet. Find the perpendicular from the centre, on a chord, 8 feet long.

16. Two parallel chords in a circle are 6 inches and 8 inches long, and 1 inch apart. Find the diameter.

17. The span of a bridge, the form of which is an arc of a circle, being 96 feet and the height 12 feet, find the radius.

18. The diameter of a circle is 12 feet ; find the side of a square inscribed in it.

19. Find the length of the minute hand of a clock, the extremity of which moves over an arc, 5 inches in length, in 3¼ minutes.

20. A perfectly flexible rope of 2¾ inches diameter is coiled closely upon the deck of a ship and there are 24 complete coils. What is the length of the rope ?

————:0:————

6

SECTION 5.

MENSURATION CONTINUED. AREAS. RECTANGLES. TRIANGLES. QUADRILATERALS. CIRCLES.

20. (*a*) The area of a *rectangle* is the product of its base by the height ; or, of any two of its adjacent sides, i. e., length and breadth.
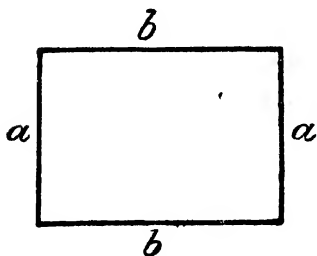
If a and b be the sides, area $= a.b$ *

or, $a = \dfrac{\text{area}}{b}$ and $b = \dfrac{\text{area}}{a}$.

That is, each side is area divided by the other side.

(*b*) The area of a *square* is the square of the length of a side.

Fig. 63

Thus if a be the side, area $= a^2$;

or $a = \sqrt{\text{area}}$.

That is, the length of a side of a square is the square root of its area.

Fig. 64

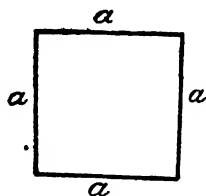
* The product of a units of length and b units of length is taken to be $a.b$ square units of area, by calling the area of a square, of 1 unit of length, broad, and 1 unit of length, long, the unit of area.

(c) The area of a parallelogram is the product of its base and its height, or perpendicular distance between the base and its parallel side.
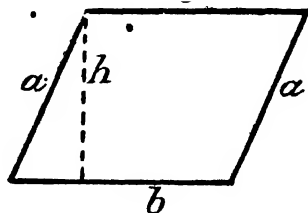

Fig. 65

If b be the base and h, height on b, area $= h.b$

Also, $h = \text{area}/b$, and $b = \text{area}/h$.

[Similarly, height on $a = \dfrac{\text{area}}{a}$, where a is one of the other sides.]

(d) In case of a trapezoid, the area is the product of the mean length of the two parallel sides and the perpendicular distance between them.

Examples. 1. The area of a rectangle, whose breadth is 12 feet, is equal to that of a square, whose side is 18 feet. Find the length of the rectangle.

Area of square $= 18 \times 18$ sq. ft.

Length of rectangle $= \dfrac{18 \times 18}{12} = 27$ feet.

2. Find the cost of paving a square room, whose side is 25 feet, with marble, at Rs. 1-4-0 per sq. foot.

Area of room $= 25 \times 25$ sq. ft.

At Rs. 1. 4 as. per sq. ft., cost of paving 25×25 sq. ft.
$= $ Rs. $625 + $ Rs. 156. 4 as. $= $ Rs. 781. 4 as.

3.   Find the area of a veranda 8 ft. 4 inches by 2 ft.

Length = 8 ft. 4 inches = 100 inches.

Breadth = 2 ft. = 24 inches.

Area = length × breadth = 100 inches × 24 inches.

= 2400 sq. inches.

$= \dfrac{2400}{144}$ sq. ft. = 16 sq. ft. 96 sq. ins.

= 1 sq. yd. 7 sq. ft. 96 sq. inches.

4.   Find the cost of metalling a pathway, 4 feet wide round a bungalow, 64 feet wide, and 100 ft. long,   at the rate of 8 annas per sq. foot.
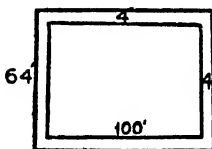
Total area of pathway

Fig. 66

= twice 100′ × 4′
    + twice (64′ + 4′ + 4′) × 4′
= 800 sq. ft + 576 sq. ft.
= 1376 sq. ft.

At 8 annas per sq. ft.,

1376 sq. ft. costs $\dfrac{1376}{2}$ or Rs. 688.

5.   What would the papering of the outside walls of the above bungalow cost, if they are 12½ ft. high, allowing 20% for doors and windows, at 2as. per sq. ft.

Allowing 20% for door etc., total area
    = 2 × 64′ × 10′ + 2 × 100′ × 10′ = 3280 sq. ft.

Total cost at 2 as. per sq. ft. = $\dfrac{3280}{8}$ = Rs. 410.

6.   Reduce (1)   21 sq. yards to sq. inches and (2) 1 sq. bigha to sq. yards.

(1)　21 sq. yds. = 21 yds. × 1 yd.

$$= (21 \times 3) \text{ ft.} \times 3 \text{ ft.}$$

$$= (21 \times 3 \times 12) \text{ ins.} \times (3 \times 12) \text{ ins.}$$

$$= 27216 \text{ sq. ins.}$$

Or tables could be used to get the result directly.

(2)　1 sq. bigha = 80 cubits × 80 cubits

$$= 40 \text{ yds} \times 40 \text{ yds.}$$

$$= 1600 \text{ sq. yds.}$$

7. Find the perimeter of a square, whose area measures 2500 sq. ft.

One side of the square = $\sqrt{2500}$ = 50 ft.

Perimeter, or sum of 4 sides = 4 × 50 = 200 ft.

8. The base of a parallelogram is 5 yds. 2 ft. Its height from the base is 3 yards 1 foot. What is its area ?

Base = 5 yds. 2 ft. = 17 ft.

Height = 3 yds. 1 ft. = 10 ft.

Area = 170 sq. ft. = 18 sq. yds. 8 sq. ft.

9. The depth of a tank is 10 feet. The bottom is 120 feet wide and the water surface, 180 feet wide. If the side slopes are straight, find the least area of the cross section of water in a vertical plane.

Mean length of 2 parallel sides = $\dfrac{180 + 120}{2}$ = 150 it.

Area = 150 × 10 = 1500 sq. ft.

EXERCISE 20.

1. Find the area, when the length and breadth are—

　(a)　7 yds. and 3 ft.

　(b)　7 yds. 1 ft. and 4 ft. 6 in.

　(c)　1 bigha (linear) and 36 cubits.

　(d)　48 yds. 2 ft. and 120 yds. 1 ft. 9 in.

2. Reduce :—

    (*a*) 1 acre to square bighas.

    (*b*) to acres, an area of 100 yds.×100 yds

    (*c*) 20 sq. bighas to sq. yards and acres.

    (*d*) to acres, an area of 1 mile×1 mile. .

3. Find the cost of carpeting a room, 16 yds.×12 yds. at Rs. 2 per sq. yard.

4. How many tiles (1′×1′ in size), will be needed to cover a roof 18 yds.×15 yds. ? If tiles sell at Rs. 30 per thousand, what would be the cost ?

5. If in metalling a road, every sq. yd. costs Rs. 2., what would be the total cost per mile of metalling a road, 40 ft. wide ?

- 6. Round a house, 15 yds long and 12 yds. wide, there is a pathway 3 ft. wide, which is to be paved with wood. If teak sells at 12 as. per sq. ft., find the total cost.

7. If marble slabs 18″ ×18″, sell at 8 as. each, find the cost of flooring a room, 15 yds ×12 yds.

8. If 1″ thick glass sheets cost Rs. 3 per sq. ft., how much would two sky-lights 1½ ft. wide and 6 yds. long each, cost ?

9. Find the total cost of covering 36 window panes, 2′×1′ each, if glass sheets, 1 yd.×1 yd. sell at 10 as. each.

10. Find the rent at Re. 1/4 per katha, of 15 acres.

11. How many square yards are there in a trapezoid, the parallel sides of which are 157·6 metres and 94 metres and the perpendicular distance between them, 72 metres. [1 metre=39 37 inches.]

12. Required the depth ot a ditch, the transverse section of which is a trapezoid, area 146·25 : breadth at top, 20 ; side slopes, 3 to 1 and 2 to 1.

13. A ditch is 30 feet wide at top and 18 feet at bottom. The earth excavated from it is formed into a bank, 28 feet wide at top and 38 feet at bottom, and 10 feet high. What is the depth of the ditch ?

14. The area of the two side walls of a rectangular room is 806 sq. feet ; the area of the two end walls, 546 sq. feet. Find the dimensions of the room.

21. (*a*) The area of a triangle is half the product of its base by the height.

If *b* be the base and $\overset{\bullet}{h}$, height of any triangle,

$$\text{area} = \tfrac{1}{2}\,(b \times h).$$

Also, $b = \dfrac{2\ \text{area}}{h}$, and $h = \dfrac{2\ \text{area}}{b}$.

When the sides *a*, *b*, *c*, only, of a triangle are given, the area is obtained as follows :—

First find $s = \dfrac{a + b + c}{2}$

Then, $(s - a)$, $(s - b)$, $(s - c)$

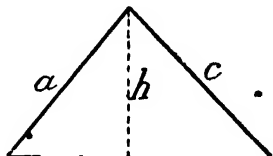$\text{Area} = \sqrt{s\ (s - a)\ (s - b)\ (s - c)}$

Fig. 67

(*b*) The cases of quadrilaterals follow from the case of triangles, as a quadrilateral can be divided into two triangles, by a diagonal. Hence the area of a quadrilateral is the sum of the areas of these two triangles.

Thus, if **BD** divides the quadrilateral **ABCD** into triangles **ABD** and **BCD**, and **CE** and **AF** are perpendiculars to **BD** from **C** and **A**,
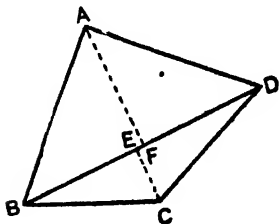
Area **ABCD**

$= \text{Areas } (\mathbf{ABD} + \mathbf{BCD})$

$= \tfrac{1}{2}(\mathbf{AF.BD}) + \tfrac{1}{2}(\mathbf{CE.BD})$

$= \tfrac{1}{2}\ \mathbf{BD}\ (\mathbf{AF} + \mathbf{CE})$

Fig. 68

That is, the area is the half

product of a diagonal and the sum of the perpendiculars on it from the opposite corners.

(c)   In case of a parallelogram, offset (art. 17, b) $\mathbf{AF = CE}$ and the area = diagonal × offset on diagonal.

(d)   In case of a rhombus, the sides are further equal and as the diagonals bisect each other at right angles ; area = $\frac{1}{2}$ product of diagonals.

(e)   Area of a quadrilateral, whose diagonals cut each other at right angles, is also given by the same formula, that is, $\frac{1}{2}$ product of diagonals.

(f)   When the sides and a diagonal of a quadrilateral are given, the area of 2 triangles into which the quadrilateral is divided by the diagonal, can be found by the application of the formula :

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Examples. 1.   The base of a triangle is 3 ft. 6 ins. and its area is 7 sq. ft.   What is its height ?

$$\text{Height} = \frac{2 \text{ area}}{\text{base}} = \frac{2.7 \text{ sq. ft.}}{3\frac{1}{2} \text{ ft}} = 4 \text{ ft.}$$

2.   Find the area of an equilateral triangle, whose perimeter is 12 ft.

Side $= \frac{12}{3} = 4$ ft.

Perpendicular $= 4 . \dfrac{\sqrt{3}}{2}$ ft.

Area $= \frac{1}{2} . 4 \times 4 \times \dfrac{\sqrt{3}}{2} = 4\sqrt{3} = 6 \cdot 928$ sq. ft. nearly.

Or, $s = \frac{12}{2} = 6$; $s - a = s - b = s - c = 2$;

area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{6.2.2.2.} = 4\sqrt{3}$

$= 6 \cdot 928$ sq. ft. nearly.

3. The sides of a triangle are 7, 8, 9 feet respectively. Find its area and perpendicular to the 9 feet side.

$$s = \frac{7+8+9}{2} = 12 \text{ ft.}$$

$\left. \begin{array}{l} s-a=5 \text{ ft.} \\ s-b=4 \text{ ft.} \\ s-c=3 \text{ ft.} \end{array} \right\}$
· Area $=\sqrt{12.5.4.3}=12\sqrt{5}$ ft

Perpendicular on 9 ft. side $=\dfrac{2 \times 12_1\sqrt{5}}{9}$

$= \frac{8}{3}\sqrt{5} = 5\text{·}92$ ft. nearly.

4. A diagonal of a parallelogram measures 5 yards and the area of the parallelogram is 20 sq. yds. Find the length of the perpendicular on the diagonal from a corner.

Area of $\frac{1}{2}$ parallelogram $= 10$ sq. yds. $=$ area of triangle.

Height of triangle $= \dfrac{2 \text{ area}}{\text{base}} = \dfrac{2.10 \text{ sq. yds.}}{5 \text{ yds.}} = 4$ yds.

EXERCISE 21.

1. The diagonals of a rhombus are 6 ft. and 8 ft. Find the area.

2. The diagonals of a rhombus are 72 and 96; find the area and the lengths of its sides.

3. Each side of a rhombus is 36 feet and one diagonal is 18 feet. Find the area of the rhombus. .

4. The area of a mat in the form of a rhombus is 8 sq. yards and the perimeter is 36 feet. Find its perpendicular breadth.

5. The side of a rhombus is 20, and its longer diagonal is 34·64. Find the area and the other diagonal.

6. · The diagonals of a rhombus are 4 ft. and 1 ft. 2 inches. Find the sides and the area.

7. The sides of a triangle are 25, 39 and 56 feet respectively. Find the perpendicular from the opposite angle on the side of 56 feet.

8. The area of an acute angled triangle of 336 sq. feet and the sides are 26 ft. and 30 feet.   Find the base.

9. The area of an equilateral triangle is 25 sq. ins. Find its perimeter.

10. The sides of a triangle are 68, 75 and 77 feet respectively. Find the length of the perpendicular on 77 feet side, from the opposite angular point

11. The sides of a triangle are 7, 24 and 25 feet respectively. Find the area.

12. The sides of a triangle are 143, 407 and 440 yds. respectively. Find the rent of the field at £2. 3s. per acre.

13. An equilateral triangle measures 1 acre. Find the length of a side in feet.

14. The sides of a triangle are in the ratio of 13, 14 and 15 and the perimeter is 50 yards. Find the area.

15. The sides of a triangular field are 191, 245 and 310 feet ; find the area in acres.

16. What is the side of an equilateral triangle, which has as many square yards in its area as lineal yards in its periphery.

17. A garden containing 1 acre, is in the form of a right-angled isosceles triangle A walk passing round it at 6 feet from the boundary wall occupies one-fourth of the whole garden. Find the width of the walk.

18. The base of a triangular field is 1210 yards, and the height is 496 yards ; the field is let for £248, a year. Find at what price per acre, the field is let.

19. Find the side of an equilateral triangle, whose area is 5 acres (give the answer in feet).

20. A triangular field, whose sides measure 375, 300, and 225 yards. is sold for £8500. Find the price per acre.

21. In a place where land cost £40 an acre, a triangular field, of which one side measured 302 yards 1 foot 6 inches, was bought for £300   What was the height of this triangle in yards ?

22.   The sides of a triangle are 17, 15 and 8 inches respectively. Find the length of the straight line joining the middle point of 17 ins. side to the opposite angle.

23.   The sides of a triangle are 25, 101, 114. Find the two parts into which the longest side is divided by the perpendicular from the opposite angle.

24.   Find the area in acres of a field **ABCD** : **AD** = 220 yards, **BC** = 265 yard, **AC** = 378 yards, and the perpendiculars from **D** and **B** meet the diagonal in **E** and **F**, so that **AE** = 100 and **CF** = 70 yards.

25.   Calculate the area of a trapezoid, the sides of which, taken in order, are 13, 11, 15, and 25, with the second, parallel to the fourth.

26.   One diagonal of a quadrilateral which lies outside the figure is 70 feet, difference between the perpendiculars upon it from angular points, is 16 feet. Find the area.

27.   Find the area of a quadrilateral **ABCD**, given **AB** = 30 inches, **BC** = 17 inches, **CD** = 25 inches, **DA** - 28 inches, **BD** = 26 inches.

28.   How many square yards are contained in a quadrilateral, one of its diagonal being 60 yards, and the perpendiculars upon it from angular points, 12·6 and 11·4 yards

29.   In a trapezium **ABCD**, **AB** = 345, **BC** = 156, **CD** = 323, **DA** - 192 ; the diagonal **AC** = 438. Find the area.

30   A railway platform has two of its opposite sides parallel, and its other two sides equal ; the parallel sides are 100 and 120 feet. respectively, and the equal sides are 15 feet each. Find its area.

————:o:————

22. (*a*) Area of a circle can be found by multiplying the square of the radius by π, or 3·1416 or $\dfrac{22}{7}$

Thus, if *r* be the radius **OA** (Fig. 69), and *d*, the diameter **AC**, of the circle **AEC**, area of circle = $\pi r^2 = \dfrac{\pi d^2}{4}$

Also, if the area of a circle be given, the radius can

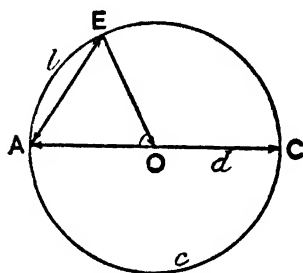be found out by dividing it by π and finding the square
root of the quotient.

Fig. 69

That is, $r = \sqrt{\dfrac{\text{area}}{\pi}}$

Also since circumference,

$$c = 2\pi r,$$

the area of the circle

$$= \pi r^2 = \frac{2\pi r.r}{2} = \frac{cr}{2}$$

$$= \tfrac{1}{2} \text{ product of radius}$$
$$\times \text{ circumference.}$$

$$= \frac{\pi d^2}{4} = 0.7854 \times (\text{diameter})^2$$

$$= \frac{c^2}{4\pi} = \frac{c^2}{12.56} = 0.07958 \ (\text{circumference})^2$$

The area of an ellipse of axes $2a$ and $2b$ is equal to $\pi\, ab$.

(b)   The area of a sector is equal to half the product
of the length of its arc and its radius.

That is, area $= \dfrac{lr}{2}$, where l is the length of arc **AB**,

$$= \frac{\text{angle in degrees at the centre}}{360} \cdot 2\pi r \times \frac{r}{2}$$

Conversely, the angle at the centre can be found from
the radius or the area.

(c)   The area of a segment of a circle is the differ-
ence between the area of its sector and the triangle
formed by the radii and the chord.

That is, area of segment $= \dfrac{lr}{2}$ – area of triangle **AOB**.

(d)   The area of a circular ring is obtained by sub-
tracting the area of the smaller circle from the bigger.

If **R** and *r* be the radii, area = $\pi R^2 - \pi r^2$.

(*e*)   The area of a triangle

= radius of inscribed circle × semi-perimeter of triangle,

$$= \frac{\text{product of sides of triangle}}{4 \times \text{radius of circumscribed triangle}}$$

(*f*)   The area of a quadrilateral of sides *a*, *b*, *c*, *d*, inscribed in a circle

$$= \sqrt{(s-a)\,(s-b)\,(s-c)\,(s-d)},$$

$$\text{where } s = \frac{a+b+c+d}{2}.$$

————:o:————

Examples 1. Find the area of a circle of (*a*) radius 7 inches, (*b*) diameter, 7 inches, (*c*) circumference 12·56 inches.

Find the area of an ellipse of 5″ and 7″ semi-axes.

(*a*) Area = $\pi 7^2 = \frac{22}{7} \times 49 = 154$ sq. inches.

(*b*) Area = $\frac{\pi 7^2}{4} = 38\frac{1}{2}$ sq. inches,

(*c*) Area = $\frac{(12·56)^2}{12·56} = 12·56$ sq. inches.

Area of ellipse = $\pi . 5 . 7 = \frac{22}{7} \times 5 \times 7 = 116$ sq. in.

2. If the area of a circle be 33 sq. inches, find the (*a*) radius, (*b*) diameter, (*c*) circumference of the circle.

(*a*) radius, = $\sqrt{\frac{33}{\pi}} = \sqrt{33 \times \frac{7}{22}} = \sqrt{\frac{21}{2}} = 3·24$ inches nearly.

(*b*) Diameter = $2 \times 3·24 = 6·48$ inches nearly.

(*c*) Circumference = $\sqrt{33 \times 4\pi} = 20·3$ inches nearly.

3. If an arc of a circle of 2 inches radius, subtends

an angle of 60° at the centre, find the area of the sector and of the segment.

Length of arc = $\frac{60}{360} \times 2 \times \frac{22}{7} \times 2 = 2\cdot1$ inches nearly.

Area of sector = $2\cdot1 \times \frac{2}{2} = 2\cdot1$ square inches.

Area of segment = $2\cdot1 - \frac{1}{2}\sqrt{3\cdot2} = 0\cdot3$ sq. inches nearly.

4   From a circle of 8 inches radius, another of 6 inches radius, is cut off.   Find the area of the space left.

Area of bigger circle = $\pi 8^2$

Area of smaller circle = $\pi 6^2$

Area of ring = $\pi (8^2 - 6^2) = \frac{22}{7} \times 28$

= 88 sq. inches.

5.   The sides of a triangle are 6, 7, and 9 feet respectively.   What are the radii of the inscribed and circumscribed circles ?

Area of triangle = $\sqrt{11 \times 5 \times 4} \times 2 = 21$ sq. ft. nearly

Radius of inscribed circle = $\frac{21}{11} = 1\cdot9$ feet nearly.

Radius of circumscribed circle $= \frac{6 \times 7 \times 9}{4 \times 21} = \frac{9}{2}$

$= 4\frac{1}{2}$ ft. nearly.

6.   The sides of a quadrilateral are 75, 75, 100 and 100 ft. respectively, and it can be inscribed in a circle. Find its area.

$s = \dfrac{75 + 75 + 100 + 100}{2} = 175.$

$s - 100 = 75$

$s - 75 = 100$

Hence area = $\sqrt{75 \times 75 \times 100}$

= 7500 sq. ft.

7.   Find the angle subtended at the centre of a circle of radius 4 inches, by an arc, $3\frac{1}{7}$ inches long.

Angle at the centre = $360 \times \dfrac{l}{2\pi r} = 45$ degrees nearly.

EXERCISE 22.

1. Assuming that the circumference of a circle is $3\frac{1}{7}$ times the diameter, find the circumference of the circle whose area is 1386 sq. ft.

2. Find in yards, correct to 3 places of decimals, the radius of a circle which encloses one acre.

3. The area of a circle is 385 acres. Find its circumference.

4. The radii of 2 circles are 6 and 8 feet respectively. Find the radius of a circle, whose area is equal to the sum of the areas of the two circles.

5. A road runs round a circular grass plot : the outer circumference is 500 yards ; the inner circumference is 300 yds. Find the area of the road.

6. A circular grass plot, whose diameter is 40 yards, contains a gravel walk 1 yard wide, running round it, one yard from the edge : find what it will cost to turf the grass plot at $4d$ per sq. yard.

7. Find the area of a gravel path, 4 feet wide, round a circular plot, whose diameter is 55 yards.

8. In cutting 4 equal circles, the largest possible, out of a piece of cardboard 10 inch square, how many square inches must necessarily be wasted ?

9. If a circle has the same perimeter as a triangle, the circle has the greater area ; verify this statement in the case where the sides of a triangle are 9, 10 and 17 feet.

10. The radius of the inner boundary of a ring is 14 inches ; the area of the ring is 100 sq. inches ; find the radius of the outer boundary.

11. (a) The area of a circle is 50 sq. yds. Find the radius.

(b) The semi-axes of an ellipse are 7 and 9 ins. Find the area.

12. The width of a circular walk is 4 feet, and the length of the line which is a chord of the outer circumference and a tangent to the inner circumference is 20 feet ; find the area of the walk.

13.   Find the area of the sector of a circle with radius 35 feet, the angle of the sector being 15°.

14.   A chord of a circle subtends an angle of 60° at the centre ; if the length of the chord be 100, find the areas of the two segments into which the chord divides the circle.

15.   The length of the arc of a circle is 14 feet ' and the radius is 10 feet ; find the area of the sector.

16.   Find the area of a sector, where the radius is 50 feet and the length of arc, 16 feet.

17.   Find the area of a segment of a circle, whose radius is 6 feet, the chord of whose arc is 8 feet.

18.   Find the area of *zone* of a circle, contained between the parallel chords, whose length are 96 and 60 inches, and their distance apart, 26 inches.

19.   Find the area of the zone of a circle, whose diameter is 20, the parallel chords being 12 and 16 long, and both on the same side of diameter.

20.   The lengths of the hour and minute hands of a clock are 10 and 13 inches respectively · find the difference between the areas of the sectors described by the hands, between 11 hours 48 minutes and 12 hours 14 minutes.

21.   Given the chord, 20 feet, and height, 4 feet, of an arc of a circle, find the area of the segment.

22.   The chord of a sector is 6 inches, the radius is 9 inches ; find the area of the sector.

23.   Find the area of a sector and of the segment of a circle, whose chord is 24, and height 6.

24.   The diameter of a rupee is $1\frac{1}{4}$ inches ; if three of these coins be placed on a table, so that the rim of each touches two others, it is required to find the area of the unoccupied space between them.

25.   Find the area of a segment of a circle whose radius is 12 and chord, 16.

26.   What is the area of the sector of a circle whose arc of 24 degrees, measures 10 feet ?

27. The radius of a circle is 15 feet; two parallel chords are drawn on the same side of the centre, one subtending an angle of 60° at the centre and the other, an angle of 120° , find the area of the zone enclosed between the chords.

28. The four sides of a quadrilateral inscribed in a circle are 80, 60, 50, and 86 feet ; required the area.

29. **ABCD** is a quadrilateral, right-angled at **B** and **D** ; also **AB**=36 chains, **BC**=77 chains, **CD** - 68 chains ; find the area.

30. **AC** is the diameter of a circle and a diagonal of the inscribed quadrilateral **ABCD** ; given **AB**=30, **BC**=40, **CD**=10. find **AD** and the area of the quadrilateral.

31. The radius of a circle inscribed in an equilateral triangle is 10 feet , find the area of the triangle

32. Find the diameter of the circle round a triangle, whose sides, are 123, 122 and 49.

33. The sides of a triangle are $2\frac{1}{4}$, 3 and $3\frac{3}{4}$ feet; find in inches, the radii of the inscribed and circumscribed circles.

34. Find the length of a side of an equilateral triangle inscribed in a circle, 8 inches in diameter.

35. Three sides of a triangle inscribed in a circle are 120, 160 and 180 feet respectively ; find the difference between the areas of the circle and the triangle.

36. Find the diameter of the circle circumscribing a triangle, the sides of which are 68, 285 and 293 feet respectively.

37. Given a circle of radius 1 foot ; find to 3 places of decimals, the sides of an equilateral triangle inscribed in it.

38. Two sides of a triangle are 85 and 154 ft. respectively, and the perimeter is 324 feet ; find the diameter of the circle described round the triangle.

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23. (a)   The area of a regular polygon can be found by dividing it into equal triangles, by joining the central point to each of the angular points and multiplying the area of a triangle by the number of sides. The area of a triangle, it will be noted, is half the product of a side of a polygon, into the altitude of the triangle, which is

(1)   the radius of the inscribed circle, or,

(2)   the square root of (square of the radius of the circumscribed circle minus square of half a side).

Thus, if n be number of sides, a, length of a side,

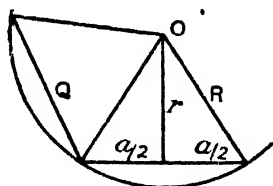area $\dfrac{n.\,a.\,r}{2}$, when r is radius of inscribed circle, or,

Fig. 70

$$= \dfrac{na\sqrt{R^2 - (\tfrac{1}{2}a)^2}}{2},$$

where **R** is radius of the circumscribed circle.

In case of a Hexagon,

$$\text{area} = \dfrac{3a^2\sqrt{3}}{2},$$

since $r = \dfrac{a\sqrt{3}}{2}$, or since $R = a$.

In an Octagon, area $= 4.a.a.\left(\dfrac{1+\sqrt{2}}{2}\right) = 4\cdot8a^2$, [§ 19, iii]

since altitude of component triangle $= \dfrac{a}{2} + \dfrac{a}{\sqrt{2}}.$

[Ex. 28, p. 72].

In a Dodecagon, the altitude is $a^2\ (2+\sqrt{3}) - \dfrac{a^2}{4}$,

R² being $a^2(2+\sqrt{3})$,   [§ 19, iv].

Area is altitude $\times a \times b$.

(*b*) The areas of irregular rectilinear figures are found by the method of base line and offsets, as outlined in article 21 (*b*), in case of a quadrilateral. One base line is often enough, but if more lines are found convenient, areas outside or inside the base lines have to be added to, or subtracted from, the area bounded by the base lines.

(*c*) The areas of irregular figures may be found out by drawing them on square paper. The small squares on the paper are of known dimensions but near the boundary, the figure will cut the squares irregularly. In such cases, the squares, of which more than half is included within the figure, are counted, while others are neglected, so that an approximate sum of the total area is obtained.

(*d*) The Simpson's rule, already explained in article 15, gives the areas of irregular figures, as follows :—

area = product of $\dfrac{\text{common distance}}{3}$

   and sum of

   (1) first ordinate plus last ordinate,
   (2) $2 \times$ sum of remaining odd ordinates,
   (3) $4 \times$ sum of even ordinates.

(*e*) Areas of similar figures are as the squares of their corresponding sides.

Examples. 1. Find the area of a regular hexagon of 4 inches sides.

Each of the 6 equal triangles, into which the hexagon can be divided, has a base = 4 inches.

   and altitude $= \dfrac{4\sqrt{3}}{2} = 2\sqrt{3} = 3\cdot6$ inches nearly.

Hence area $= 6 \times \frac{1}{2} \times 4 \times 3\cdot6 = 43\cdot2$ sq. inches.

In this case, the altitude of a component triangle, or radius of inscribed circle can be deduced from the side, as has been done.

2. Find the area of a regular octagon of 4 inch sides.

Here, altitude $= 4 \times 1{\cdot}2 = 4{\cdot}8$ inches nearly. [Ex. 28, p.72]

Hence area $= \dfrac{8}{2} \times 4 \times 4{\cdot}8 = 76{\cdot}8$ sq. inches.

3. **AOCNBM** is an irregular figure as shown :
Diagonals **AB** = 6, **AC** = 7 and **BC** = 8, respectively.
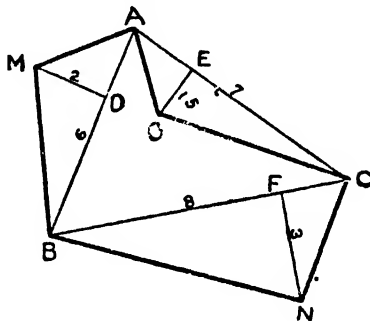
Fig. 71

Find the area, when perpendiculars or offsets on base lines **AC, CB, BA**, measure as follows :—

$$MD = 2, \quad OE = 1\tfrac{1}{2}, \quad FN = 3.$$

Area of traingle **AOC** $= \tfrac{1}{2}(7 \times 1\tfrac{1}{2}) = \tfrac{21}{4} = 5{\cdot}25$

,,   **AMB** $= \tfrac{1}{2}(6 \times 2) = \tfrac{12}{2} = 6$

,,   **BNC** $= \tfrac{1}{2}(8 \times 3) = \tfrac{24}{2} = 12$

,,   **ABC** $= \sqrt{\tfrac{21}{2} \times \tfrac{9}{2} \times \tfrac{7}{2} \times \tfrac{5}{2}} = \tfrac{21}{4}\sqrt{15} = 20{\cdot}5$

Required area $=$ **ABC + AMB + BNC – AOC**

$= 20{\cdot}5 + 6 + 12 - 5{\cdot}25$

$= 33{\cdot}25$

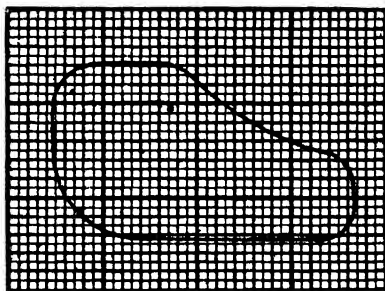4. Find the area of the figure on square paper below.

Fig. 72

The side of a square between thick lines = 0'5 in.

The area   „   „   = 0'25 sq. in.

The side of a small square = 0'05 in.

The area   „   = 0'0025 sq. in.

Number of big squares = 1

Area   „   = 0'25 sq. in.

Number of small squares, (covered, more than half)
= 330

Area of small squares = 330 × 0'0025 = 0'825 sq. in.

Total area :   Big square—0'25 sq. in.

Small squares—0'825 sq. in.

Total area = 1'075 sq. in.

5. The ordinates of a curve are 2, 3, 4, 6, 5, 3, 2 inches, the common distance being 1 inch. Find the area of the curve.

$$\text{Required area} = \frac{1}{3}\left\{4 + 2 \times 9 + 4 \times 12\right\}$$

$$= \frac{70}{3} = 23\frac{1}{3}\text{sq. inches.}$$

6. What will be the size of the map of a district, which contains 676 sq. miles, on a scale of 4 miles to the inch ?

$$\frac{\text{Required size}}{676 \text{ sq. miles}} = \frac{(1 \text{ inch})^2}{(4 \text{ miles})^2}, \text{ or,}$$

required size $= 42\frac{1}{2}$ sq. inches.

EXERCISE 23

1. Calculate to three decimal places, the area of a regular hexagon, each of whose sides is equal to 10 feet.

2. Find the area of a regular octagonal field, each of whose sides measures 5 chains (give the result in acres, roods, etc).

3. An ornamental grass plot is in the shape of a regular hexagon, each side 100 feet ; within the plot and along its sides, a foot path is made, 4 feet wide all round ; find the area of the grass plot left within.

4. A regular decagon is inscribed in a circle, the radius of which is 10 inches. Find the area.

5. Compare the areas of an equilateral triangle, a square and a regular hexagon of equal perimeter.

6. The area of a regular octagon is 51 sq. yards. Find the length of its sides.

7. Find the area of a regular octagon, whose side is 20 feet.

8. Find the side of a regular octagon inscribed in a square, the area of which is $6 + 4\sqrt{2}$ sq. feet

9. A regular dodecagon is inscribed in a circle, of which the radius is 3 inches. Find the area of the polygon in sq. feet.

10. The radius of a circle is 12 feet. Find the length of the side of a polygon of 16 sides, inscribed in it. Calculations to be made to 3 places of decimals.

11.   Make a rough sketch and find the area of a field **ABCD** from the following measures taken in links and find the length of the perpendicular from **A** on **CD**.

> **BM**, perpendicular from **B** on **AC** = 400
>
> **DN,**   „   ,   „   **D** on **AC** = 300
>
> **AM** = 300, **AN** = 400, **AC** = 625.

12.   The sides of a five-sided figure **ABCDE** are as follows :
AB = 25 ft,   **BC** = 29 ft.,   **CD** = 39 ft.,   **DE** = 42 ft., and **EA** = 27 ft. ;
also, **AC** = 36 ft , and **CE** = 45 ft. :   Find its area.

13.   In the pentagonal field **ABCDE**, the length of **AC** = 50 yards and the perpendiculars from **B**, **D** and **E** upon **AC** are 10, 20 and 15 yards , the distances from **A** of the feet of the perpendiculars from **D** and **E**, being 40 and 10 yards · Find the area.   •

14.   What is the content of the eight-sided figure **ABCDEFGH**, the diagonal **AE** being taken as a base, and the perpendiculars drawn from the angular points to **AE**, being **B**b, **C**c, **D**d, etc., and when the lengths of the perpendiculars above the diagonal are **B**b = 294, **C**c = 142·5 .   **D**d = 224 ; and those below the diagonal are **F**f = 121, **G**g = 195·5,   **H**h = 142 and the intercepted breadths are **A**h = 44·5 ; hb = 124·25, bg = 80, gc = 41, cd = 130 5, df = 50, fe = 52 5.

15.   Draw the figures in examples 12, 13, and 14 on squared paper on a suitable scale and obtain the areas.

16.   Take the figures in example 15 to represent yards and draw the figure on squared paper on a suitable scale.   Find the area.

17.   Apply Simpson's rule to find the area of a plot of a land having the following dimensions :

> Ordinates, 2, 7, 18, 38, 70 feet :
> Common distance, 33 feet.

18.   Find by Simpson's rule, the area of a figure, whose ordinates are 9, 11, 13, 12, 10, 13, 15, 17, 14, 12, 7 ft. ; base, 73 feet.

19.   Apply Simpson's rule, to find the area of a section, the heights of which above the Railway level at intervals of 30 feet, are 2, 10, 15, 20, 30, 25, 17·5, 10, 3 feet.

20. Find the dimensions of a triangle similar to one, whose dimensions are 50, 60, and 80 feet, but which shall contain three times the quantity.

21. A board is 12 inches wide at one end and 9 inches at the other end, and its length is 8 feet : how far from the broad end must it be sawn across, so as to divide the plank into two equal portions.

22. A circular hole is cut in a circular plate, so that the weight may be reduced by one-third : find the diameter of the hole as a fraction of that of the plate.

23. A drawing is copied to a scale, one half as large again, as the original scale : in what ratio is the surface augmented ?

24. On a map drawn to the scale of $\dfrac{1}{10000}$, the sides of a rectangular field are 0·65 and 0·72 ins. ; find the area of the field in acres and the length of the diagonal in yards.

25. Find the scale to which a plan is drawn, 1 sq. inch representing 10 acres.

26. The sides of a triangle are 39, 52 and 65 feet, respectively : find the sides of a similar triangle of 9 times the area.

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SECTION 6.

MENSURATION—VOLUMES.

Surface areas and Volumes of solids, Prisms, Parallelopiped, Cylinder, Pyramid, Wedge, Cone and Sphere.

24. (a) A *solid* is a figure having length, breadth and thickness and is bounded by a surface or surfaces, on all sides. When the surfaces of a solid are planes, they are called *faces*. The lines of intersection of the faces are called *edges*.

A *prism* is a solid with plane faces, whose end faces (top and base) are two equal figures, lying in harallel planes and whose other faces (sides) are either (1) rectangles, when the prism is called a *right* prism ; or, (2) parallelograms, when the prism is called an *oblique* prism.

It is called triangular, rectangular, square, pentagonal, hexagonal, etc., according to the shape of the end faces.

A *cube* is a right prism having six equal square faces, contiguous faces being at right angles. The edges are equal and the length, breadth and height of a cube are the same as the edges.

Fig. 73

If a denotes the length,

area of each face $= a \times a = a^2$.

Total area of 6 faces $= 6a^2$

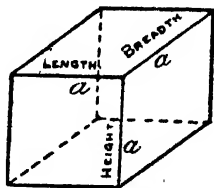Volume of cube $=$ length $\times$ breadth $\times$ height $= a^3$.

(b) A *rectangular solid* is a right prism, having 6 rectangular faces, opposite faces being equal and parallel.

If l denotes the length, b, breadth and h, height of the rectangular solid :

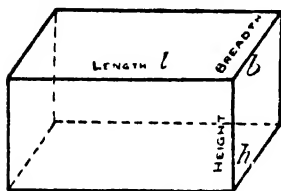Areas of 3 faces are $l \times b$, $b \times h$ and $h \times l$ respectively.

Fig. 74

Total area $= 2(lh + bh + lb)$.

Diagonal of face $lb, = \sqrt{l^2 + b^2}$

,, $bh, = \sqrt{b^2 + h^2}$

,, $lh, = \sqrt{l^2 + h^2}$

Volume $=$ length $\times$ breadth $\times$ height,

$=$ area of base $\times$ height. $= l.\, b.\, h = \sqrt{lb.\, bh.\, lh}$.

$= \sqrt{\text{product of 3 contiguous faces.}}$

Diagonal of the solid is the distance between two opposite corners, through the centre of the figure

$$= \sqrt{l^2 + (b^2 + h^2)} = \sqrt{b^2 + (l^2 + h^2)} = \sqrt{h^2 + (l^2 + b^2)}$$

There are 4 such diagonals, through the centre of the figure.

(c) An oblique prism which has 6 faces, all of which are parallelograms, is called a *parallelopiped*

Here, area of surface $=$ sum of areas of its faces.
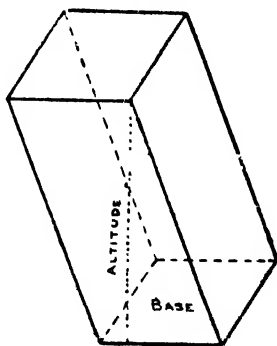
Volume $=$ area of base $\times$ altitude.

Fig. 75

The case also follows from right prisms, since prisms on the same base and between the same parallel planes, have equal volumes.

(*d*) The volume of any right prism is obtained by multiplying the area of the base by the height.

Thus, if **A** be area of the base and *h*, height, volume = **A**. *h*.

Area of surface
= perimeter of base × height
+ 2 area of base.

The case may be extended to the right cylinder, whose

Fig. 76

circular base may be considered as bounded by an infinite number of small equal sides.

The area of base = πr^2, where *r* is radius.

Volume = $\pi r^2 h$, where *h* is height of the

Fig. 77   cylinder.

Area of surface $= 2\pi r^2 + 2\pi r. \ h = 2\pi r \ (r + h)$

The case of a hollow cylinder, follows easily.

Area of angular base
$= \pi \ (R^2 - r^2)$,
where **R** and *r* are radii.

Volume of hollow cylinder
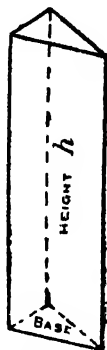$= \pi h \ (R^2 - r^2)$
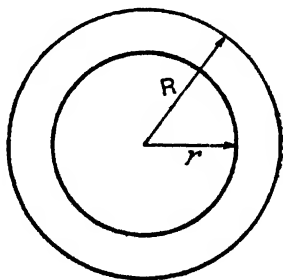$= \pi h \ (R + r) \ (R - r)$.

Fig. 78

(e)   The volume of an oblique prism is obtained by multiplying the area of cross-section, perpendicular to the edges of the side faces, by the length of the prism.
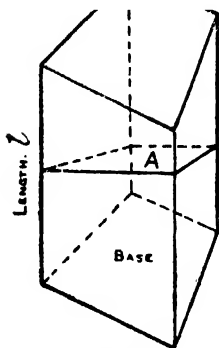

Fig. 79

Thus if **A** be the area of the cross-section, *l*, length,

volume = **A***l*.

For, it will be noticed, that if we cut the prism through the section and place the lower portion on the top' of the upper, we get a right prism of base **A** and height *l*.

The case of an oblique cylinder also follows from this.

Examples. 1. (*a*) Find the edges of a cube whose volume is 3 cubic yards, 10 cubic feet and 1944 cubic inches.

Volume
= 3 cu. yds. 10 cu. ft. 1944 cu. ins. = 157464 cubic inches.

Edge = $\sqrt[3]{157464}$ = 54 inches = 4 ft. 6 inches.

(*b*)   What is the volume of a tank of 1 ft. 6 inches edges ?

Edge   = 1 ft. 6 inches = 18 inches.

Volume = 18 × 18 × 18 = 5832 cubic inches,

= 3 cubic feet 648 cubic inches.

(c) **How many sheets of brass, 12 inches × 18 inches size, will be required to prepare a cubic tank of 1 cubic yard capacity ?**

Volume is 1 cubic yard.   Edge is 1 yard = 3 feet.

Area of surface of a face = 9 sq. feet.

Total area of 6 faces = 54 sq. ft.

Area of each brass sheet = $12'' \times 18'' = 1\frac{1}{2}$ sq. ft.

Number of sheets = $\dfrac{54}{1\frac{1}{2}} = 36$ pieces.

2. **A brick measures 9 inches × $4\frac{1}{2}$ inches × 3 inches. How many cart loads will be required to prepare a wall, 90 feet long, by 9 feet high, by 1 foot wide, if a cart carries 432 bricks ; plasters, and mortars, etc. occupying 10% space ?**

Volume of a brick = $9 \times \frac{9}{2} \times 3$ cubic inches.

Volume of bricks in a cart = $9 \times \frac{9}{2} \times 3 \times 432 \times \dfrac{1}{1728}$ cu. ft.

$$= \dfrac{9 \times 9 \times 3}{8} \text{ cubic feet.}$$

Number of cart loads = $\dfrac{\frac{9}{10} \text{ volume of wall}}{\text{volume of a cart load}}$

$$= \dfrac{90 \times 9 \times 1 \times \frac{9}{10}}{9 \times 9 \times \frac{3}{8}} = 24$$

3. **A rectangular solid has the following dimensions : Length, 5 feet ; breadth, 4 feet ; width, 3 feet.   Find the areas of its faces, total surface area, diagonals of each face, volume and diagonal of the solid.**

Areas of faces : $5 \times 4 = 20$ sq. ft.   for 2 faces.

$4 \times 3 = 12$ sq. ft.   „

$5 \times 3 = 15$ sq. ft.   „

Total surface area $= 2\,(20 + 12 + 15) = 94$ sq. feet.

Diagonals of faces : $\sqrt{5^2 + 4^2} = \sqrt{41} = 6.2$ ft. nearly.

$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5 \text{ ft}$$

$$\sqrt{5^2 + 3^2} = \sqrt{34} = 5.9 \text{ ft. nearly.}$$

Volume $= 5 \times 4 \times 3 = 60$ cu. ft.

Diagonal of solid $= \sqrt{5^2 + (4^2 + 3^2)}$

$$= \sqrt{50} = 7 \text{ ft. nearly.}$$

4. An oblique prism has a rectangular base, 4 inches long by 3 inches broad, and a slant height of 5 inches, so that the projection of the top, on the plane of the base, forms with the base, a rectangle, 8 inches × 3 inches   Find the surface area and the volume.

Altitude $= \sqrt{5^2 - 4^2} = 3$ inches.

Area of a vertical face (parallelogram)

$$= \text{altitude} \times \text{base} = 3 \times 4$$

Area of slant face (rectangular) $= 5 \times 3$.

Total area = areas of 2 end faces plus areas of 2 vertical faces plus areas of 2 slant faces

$$= 2 \times 4 \times 3 + 2 \times 3 \times 4 + 2 \times 5 \times 3$$

$$= 78 \text{ sq. inches.}$$

Volume = area of base × altitude

$$= 3 \times 4 \times 3$$

$$= 36 \text{ cu. ins.}$$

5. A wall 28 feet deep and 6 feet diameter has to be lined with brick-work, $1\frac{1}{2}$ feet thick. Find the cost at Rs. 44/- per 100 cubic feet.

Area of section of brick-work $= \pi\{3^2 - (\frac{1}{2})^2\}$

$$= \pi\,\frac{27}{4}\ \text{sq. ft.}$$

Volume of brick-work $= \dfrac{22}{7} \times \dfrac{27}{4} \times 28$ cu. ft. $= 594$ cu. ft.

Cost $= \dfrac{594}{100} \times 44 = $ Rs. 262 nearly.

EXERCISE 24.

1 Three cubes of metal, whose edges are 3, 4 and 5 inches respectively, are melted and are formed into a single cube ; if there be no waste in the process, show that the edge of the new cube will be 6 inches.

2 Find the length of the longest rod that can be placed in a room 30 feet long, 24 feet broad and 18 feet high.

3 A box without lid is made of wood, one inch thick ; the external length, breadth and height of the box are 2 feet 10 inches, 2 feet 5 inches, and 1 foot 7 inches respectively ; find what volume the box will hold, and the number of cubic inches of wood.

4. A reservoir is 24 feet 8 inches long by 12 feet 9 inches wide ; find how many cubic feet of water must be drawn off, to make the surface sink 1 foot.

5. The diagonal of a cube is 30 inches. What is the solid content ?

6. A school room is to be built to accommodate 70 children, so as to allow $8\frac{1}{2}$ sq. feet of floor and $110\frac{1}{2}$ cubic feet of space for each child ; if the room be 34 feet long, what must be its breadth and height ?

7. Find how many bricks, of which the length, height and thickness are 9, $4\frac{1}{2}$ and 3 inches will be required to build a wall, of which the length, height and thickness are 72, 8 and $1\frac{1}{2}$ feet.

8. How many superficial feet of inch plank can be sawn out of a log of timber 20 feet 7 inches long, 1 foot 10 inches wide, 1 foot 10 inches deep ?

9. What are the cubic contents of a shaft, the mean section of which is a regular hexagon, of $2\frac{1}{2}$ feet sides, and the height, 60 feet ?

10. The trunk of a tree is a right circular cylinder, 5 feet radius and 30 feet high. Find the volume of the timber which remains, when the trunk is trimmed just enough to reduce it to a rectangular parallelopiped on a square base.

11. A cubical foot of brass is drawn into a cylindrical wire 1/40 of an inch in diameter. This wire is just long enough to go round a circular field ; find approximately the area of the field in acres.

12. A well 7½ feet inside diameter, is to be sunk 22 feet deep, with a brick lining of 13½ inches in thickness. Find—

> (a) Excavation of earthwork.
> (b) Quantity of brick-work.

13. An iron pipe is 3 inches in bore, ½ inch thick, and 20 feet long. Find its weight, supposing that a cubic inch of iron weighs 4·526 ozs.

14. A well is to be dug 5 feet inside diameter and 36 feet in depth. Find the quantity of the earth to be excavated, and the quantity of brick-work required for a lining of 10 inches in thickness.

15. Find the quantity of masonry in a well 10 feet interior diameter, 50 feet deep ; thickness of masonry ring is 18 inches. What would be the cost of constructing the masonry at the rate of Rs. 25/- per 100 cubic feet ?

16. A carriage drive is to be made round the outside of a circular park, whose radius is 585 feet ; the metalling is to be 30 feet wide and 9 inches deep : What will it cost at Rs. 6/-per 100 cubic feet?

17. The base of a certain prism is a regular hexagon ; every edge of the prism measures 1 foot : Find the volume of the prism.

18. The section of a canal is 32 feet wide at the top, 14 feet wide at the bottom, and 8 feet deep. How many cubic yards were excavated in a mile of the canal ? Also, if the surface of water be 26 feet wide, what is the depth ?

—:o:—

25. (*a*) A *pyramid* is a solid whose base is a plane rectilinear figure and whose sides are triangles, having a common *vertex* of the pyramid.

Thus the triangles **ABCDO** is a pyramid, with **ABCD** as base and the triangles **OAB**, **OBC**, **OCD**, and **ODA** as sides, **O** being the vertex. It is called a regular pyramid when the base is a regular figure.
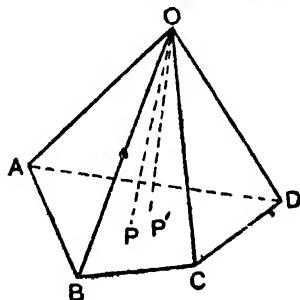

Fig. 80

The line joining the central point of the base of a pyramid to its vertex is the axis of the pyramid. The length of the perpendicular from the vertex to the base is called the *height* of the pyramid. Thus **OP′** is the axis and **OP** the height of the pyramid.

The slant height of a right regular pyramid is the length of the line joining the vertex to the middle point of the base. When the axis is perpendicular to the base the pyramid is a *right* pyramid. Otherwise it is *oblique.*

The surface area of a pyramid is the sum of the areas of the base and of the slant surface (viz. of the sides).

A pyramid is called triangular, square, pentagonal, hexagonal etc., according to the shape of the base.

8

(b) If the central point of a cube be joined to each of the angular points, 6 equal right square pyramids are formed.
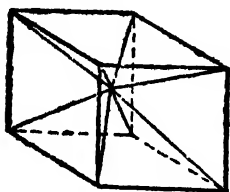

Volume of each pyramid
= $\frac{1}{6}$ volume of cube.
= $\frac{1}{6}$ base of cube × height of cube
= $\frac{1}{3}$ base of pyramid × height of pyramid.

Since the volumes of pyramids on the same base and between the same parallels are equal, the same relationship for oblique prisms also holds.

Fig. 81

For example, if each of the 4 corners of the base of a cube be joined to any of the corners of the opposite parallel face, we get a square pyramid whose volume is $\frac{1}{3}$ volume of the cube.

Generally, the volume of *any* pyramid is one-third the product of the base and height of the pyramid.

A triangular pyramid is called a *tetrahedran*. A regular tetrahedran has equal equilateral triangles for its 4 faces.

(c) The volume of a wedge as shown in fig. 82 is given as below :

If 'a' be the length, 'b' be the breadth of the base, 'e' the edge and 'h' the height of the wedge the volume will be $= \dfrac{(2a + e)\, b \times h}{}$
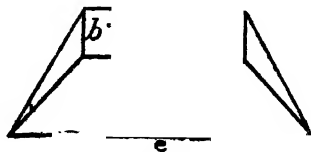
$= \dfrac{bh}{6}(2a + e).$

Fig. 82

(*d*) The case of a pyramid may be extended to the circular *cone*, whose base may be considered as bounded by an infinite number of small equal sides.

If *r* be the radius of the base and *h*, the height of the vertex,

volume $= \frac{1}{3}\pi r^2 h$.

For a right circular cone,

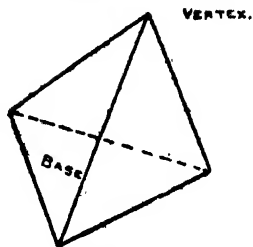surface area = area of base *plus* circumference $\times \frac{1}{2}$ slant height.

Fig. 83

Volume of a right circular cone is one-third that of a cylinder of same base and height.

A right circular cone is a figure described by the revolution of a right-angled triangle of base equal to the radius of the base of the cone and height equal to the height of the cone.

Examples. 1. Find the volume of a pyramid, whose square base has 8 ins. sides and whose height is 9 inches. If it is a right pyramid, what is the surface area ?

Area of base $= 8^2 = 64$ sq. inches.

Volume $= \frac{1}{3}. \, 64 \times 9 - 192$ sq. inches.

Half base $= 8/2 = 4$ inches.

Slant height $= \sqrt{9^2 + 4^2} = \sqrt{97} = 9\cdot8$ inches nearly.

Area of each side $= \frac{1}{2} \times 9\cdot8 \times 8 = 39\cdot2$ inches.

Total surface area $= 4 \times 39\cdot2 + 64 = 156\cdot8 + 64$

$\qquad\qquad\qquad = 220\cdot8$ inches nearly.

2. A regular tetrahedran has each edge $= 4\sqrt{3}$ inches. Find its height, surface area and volume.

Let **ABCD** be the figure. **AP,** slant height,

Fig. 84

$$= 2\sqrt{3} \times \sqrt{3}$$
$$= 6 \text{ inches.}$$

OP $= 2\sqrt{3}/\sqrt{3} = 2$ inches from triangle **OPC.**

OA, height $= \sqrt{\overline{AP^2 - OP^2}}$
$$= \sqrt{36 - 4} = \sqrt{32}$$
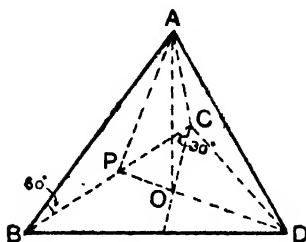$$= 5\cdot7 \text{ ins. nearly.}$$

Total surface area $= 4 \times$ area of triangle **ABC.**
$$= 4 \times \tfrac{1}{2} \times 4\sqrt{3} \times 6$$
$$= 48\sqrt{3} = 86\cdot4 \text{ sq. inches.}$$

Volume $= \tfrac{1}{3}$ area of triangle **ABC** $\times$ height
$$= \tfrac{1}{3} \times \tfrac{1}{2} \times 4\sqrt{3} \times 6 \times 5\cdot7 = 41 \text{ cu. ins. nearly.}$$

3.   The edge of a wedge is 11 inches, the length of the base is 6 inches and the breadth is 4 inches ; the height of the wedge is 4 inches.   Find the volume.

$$\text{Volume} = (2a + c)\,\frac{bh}{6} = (2 \times 6 + 11)\,\frac{4 \times 4}{6}$$
$$= 23 \times \tfrac{8}{3} = 1\tfrac{84}{3} = 61\tfrac{1}{3} \text{ cu. in.}$$

4.   Find the slant height, surface area and volume of a right circular cone, whose height is 8 inches and radius of whose base is $3\frac{1}{2}$ inches.

Slant height $= \sqrt{8^2 + (3\tfrac{1}{2})^2} = 8\cdot73$ inches nearly.

Surface area $= \tfrac{1}{2} \times 2\pi3\tfrac{1}{2} \times 8\cdot73 + \pi(3\tfrac{1}{2})^2$
$$= 96\cdot03 + 38\cdot48$$
$$= 134\cdot57 \text{ sq. inches nearly.}$$

Volume $= \tfrac{1}{3}\pi(3\tfrac{1}{2})^2 \times 8$
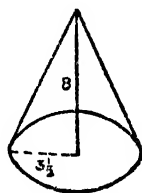$$= 102\cdot6 \text{ cu. ins. nearly.}$$

Fig. 85

EXERCISE 25.

1. Find the volume of a pyramid when its base is a regular hexagon, each side measuring 6 feet and height, 30 feet.

2. A regular hexagonal pyramid has the perimeter of its base 15 feet and its altitude 15 feet. Find its volume.

3. A pyramid on a square base has 4 equilateral triangles for its 4 other faces, each edge being 20 feet. Find the volume.

4. Find the volume of a pyramid, formed by cutting off a corner of the cube, whose edge is 20 feet, by a plane which bisects its three conterminous edges.

5. A solid is bounded by 4 equilateral triangles, a side of each triangle being 12 inches Find the volume.

6. A pyramid has for its base an equilateral triangle of which each side is 2 feet, and its slant edge is 6 feet. Find its solid content.

7. A pyramid on a square base has 4 equilateral triangles for its 4 other faces, each side being 30 feet. Find the volume.

8. Find the volume of the regular triangular pyramid, a side of its base being 6 feet, and its altitude, 60 feet.

9. A right-angled triangle, of which the sides are 3 inches and 4 inches in length, is made to turn round on the longer side ; find the volume of the cone thus formed.

10. Find the solidity of a cone, the diameter of whose base is 3 feet and its altitude is 30 feet.

11. A piece of tin having the form of a quadrant of a circle is rolled up so as to form a conical vessel ; required its content, when the radius of the quadrant is 10 inches.

12. A conical tent is required to accommodate 5 people : each person must have 16 sq. feet of space on the ground, and 100 cubic feet of air to breathe ; give the vertical height, slant height, and width of the tent.

13. Find how many gallons are contained in a vessel which is in the form of right circular cone, the radius of the base being 8 feet and the slant side, 17 feet.

14. A right-angled triangle, whose remaining angles are 60° and 30°, revolves about its hypotenuse, which is 12 inches long. Find the volume of the solid thus described.

15. The base of a wedge is a square, a side of which is 15 inches ; the edge is 24 inches and the height is 24 inches ; find the volume.

16. The edge of a wedge is 9 feet, the length of the base is 6 feet and the breadth is 3 feet ; the height of the wedge is 2 feet ; find the volume.

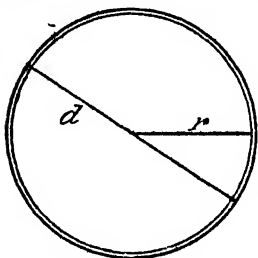**26.** A sphere is a solid figure described by a semicircle about the diameter.

Fig. 86

If r be the radius,
surface area $= 4\pi r^2$
volume $= \frac{4}{3}\pi r^3$

Example. Find the weight of a spherical metal ball of 4 ins. radius, if 1 cubic inch weighs 2 lbs. What will be the cost of guilding it at the rate of 3 annas per sq. in.

Volume $= \frac{4}{3} \times \pi 4^3 = 267\cdot84$ cu. ins.
Weight $= 2 \times 267\cdot84$ lbs. $= 535\cdot68$ lbs.
Surface area $= 4\pi 4^2 = 28\cdot16$ sq. ins.
Cost of guilding $= 28\cdot16 \times 3$as. $=$ Rs. 5. 4as. 6p. nearly.

EXERCISE 26.

1. A sphere is 36 inches in diameter ; find the area of its surface in square inches.

2. A circular room has perpendicular walls, 15 feet high, the diameter of the room being 28 feet , the roof is a hemispherical dome : find the cost of plastering the whole surface at 9d. per square foot.

3. What is the surface of a sphere whose diameter is 21 inches ?

4. Find the volume of a sphere when its surface is (numerically) equal to that of a circle, 9 feet in diameter.

5. A Cathedral has 2 spires and a dome : each of the former consists, in the upper part, of a pyramid 60 feet high, standing on a square base, of which a side is 20 feet.

The dome is a hemisphere of 40 feet radius. Find the cost of covering the three, with lead at $7\frac{1}{2}d$. per square foot. ($\pi = 3.1416$).

6. A wrought iron cylindrical boiler, 10 feet long, 4 feet in diameter, and 3/8 inch thick (inside measurements) is closed by hemispherical ends. Find the external surface.

7. A cylinder 12 feet high and 6 feet in diameter is surmounted by a cone, also 6 feet in diameter and 4 feet high ; find the radius of a hemisphere whose entire surface is equal to the united curved surfaces of the cone and the cylinder.

8. The price of a ball at $1d$., the cubic inch, is as great as the cost of guilding it at $3d$. the square inch : what is its diameter ?

9. A sphere and a cube have the same surface : show that the volume of the sphere is 1.38 times that of the cube. ($\pi = 3.1416$).

10. A sphere has the same number of cubic feet in its volume as it has square feet in its surface ; find the diameter.

SECTION 7.

SURVEYING.

Principles. Chains. Offsets. Field-book. Maps.

27. (*a*) **Surveying.** It consjsts of 'measuring and representing in a map or plan, lengths and boundaries and the positions of different objects occuring in a certain area of land. The principle of chain surveying, (where a chain is used for measurements) known as *triangulation*, depends on the problem :

"Three sides of a triangle being given, to construct the triangle."

The work, first *in the field*, consists of simply recording in the *field-book*, the measurements made of the length of straight lines, which form the triangles :

(1) base lines (bases of triangles) suitably chosen, on the land to be surveyed, and

(2) distances of various objects such as trees, buildings, roads, tanks etc., with reference to the base lines, i. e.,

(*a*) their distances (sides of triangles) from points on the base, or from the ends of the base. [Mainly used for isolated objects at a distance.]
or (*b*) their perpendicular distances (off-sets, article 17) from the base. [Commonly used.]

Additional lines called *tie* or *check* lines from the object to the middle points of the base or points near the middle, are also measured, in order to check the corret-

ness of the measurements of the triangles. Often, a number of lines has to be chosen forming rectilinear figures, other than triangles, but resolvable into trianles, involving, in fact, an extension of the system of triangulation.

Secondly, from the records in the field-book, a reduced copy of the country or map, showing various objects on it, is made to any scale.

Before the actual commencement of the survey, a clear idea of the land should be formed, to enable the surveyor to divide it into convenient triangles (*triangulation*). These triangles are marked on the ground by pegs driven on the angular points, which are called *stations*. In choosing the triangles, the following points are considered :

1. triangles should be as large as possible,
2. sides of triangles should, as far as possible, be close to the boundary of the land :
3. triangles should be equilateral, as far as practicable.

The base lines should be so chosen that the triangles, as suggested, may be conveniently constructed.

A rough hand sketch is made at this stage showing the various objects on the land, the proposed stations and base lines, their alignments, offsets and other lines.

After the triangulation of the land, measurements of the base lines and of the offsets and other lines from various objects on both sides of the lines are taken by the *chain, measuring tape,* or *offset rod.*

(b) **Chain.** They are formed of a number of equal parts, called *links*, of steel wire, so made for permananancy and convenience in carrying.

There are three kinds of chains used :

Fig. 87

1. The hundred-feet chain, divided into 100 equal links. This is used in all engineering surveys of roads, buildings, bridges, etc., where the unit *foot* is invariably used.

2. The Gunter's chain of 66 feet length, divided into 100 equal links, each measuring 7·92 inches. This is used in all land surveys, as areas are conveniently obtained from measurements by it. For, 1 acre = 43560 sq. ft. = 66 × 66 × 10 sq. ft. = 10 sq. chains.

3. The thirty-feet chain divided into 40 equal links, each equal to 9 inches, eight links making a katha (4 cubits). This is however seldom used now-a-days.

Brass pieces with notches or points attached to the chain, at equal intervals, indicate the position of the pieces by the number of notches or points·with reference to the ends of the chain, which are made of swivel handles, for convenience in pulling it.

The chain is used as follows :

The line to be measured should be clearly marked out ; this is done by placing flags at the ends, and if

the line be long, a few flags are also placed between the ends, in their line. This operation of placing flags is called *ranging the line.*

Two men are required to measure a line ; one man goes ahead, dragging the chain. He is called the *leader.* The man behind is the *follower.* He fixes one end of the chain and directs the leader to go along the line, ranged as above. The leader carries 10 arrows (steel pins, about 16 inches long, as shown in the figure), to start with, and drives one end into the ground at the end of the stretched chain. The leader

Fig. 88

then pulls the chain to its second stretched position, and drives the second arrow and so on, the follower picking up the arrows, when he arrives at them after the arrow ahead is fixed, until the 10th arrow is picked up. These arrows, thus collected, are then handed over to the leader and the process goes on, until the whole length of the line is measured.

Great care is taken to preserve the proper direction in measuring line with the chain. The leader sees carefully that the arrow he fixes, the arrow near the follower and the flag at the starting end of the line, are in the same straight line, whereas, the follower in his turn, sees that his arrow, leader's arrow and the flag at other end of the line, are in the same straight line.

The *measuring tape* is used to measure small lengths. The outer end of the tape is held by one man at one extremity of the length to be measured, while the other man opens out the tape and runs the tape over the other extremity of the length. The exact point of the tape coinciding with the extremity is then read.
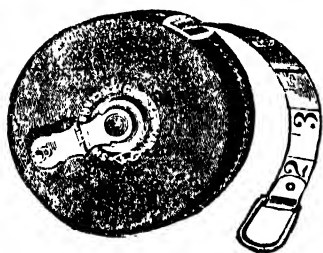
Fig. 89

The *offset rod* is a straight piece of bamboo or

Fig. 90

wood about $1\frac{1}{2}''$ diameter and 10 links long and graduated to single links, by painting alternate links. One end is sharp and mounted in iron for driving the rod into the ground.

It must be noted that as horizontal distances only are required, the chain or tape should be held horizontally, if necessary, in small lengths, when steep slopes are measured.   This is known as *stepping* and *cutting*.

N. B.   The chain is liable to the following errors :—

(1) Error in itself, specially when it is new   Also all chains are lengthened by use. They should therefore be properly tested before use, as follows :

The chain is stretched on a level ground by two stout pins at its extremities.   Two measuring staffs or offset

rods of correct and known lengths are laid down, end to end, along the chain from one pin. The first is removed keeping the second in position and placed beyond the second. The second is next removed and placed beyond the other, and so on, until the whole length of the chain is covered. If the measurement by the rods differs from that of the chain, it is corrected by taking out from, or adding, small rings to the links of the chain.

At present, almost, every town in the Province has standards for testing chains and other survey appliances, with which they can be compared for correction.

(2) Error in the method of using the chain. If the chain is stretched too tight, the rings will give, the arrows incline and the measured line will be too short. On the other hand, if it be not sufficiently tight, the measurement will be too long.

(3) Error in the uncertainty of placing the arrows, which must be driven on the ground firmly and vertically, inside the rings or handles at the ends of the chain, as far apart from each other, as practicable.

(c) **Offsets.** These have been explained in art. 17 They are of two kinds :—

1. *Right* offsets, which are perpendicular distances measured from the chain line to any object, either to the right or left of it.

2. *Oblique* offsets, which are the distances from an object to two points on the chain, representing the two sides of a triangle, of which the line between the two points, is the base.

The offsets are measured with offset rods, chain, measuring tape, etc.

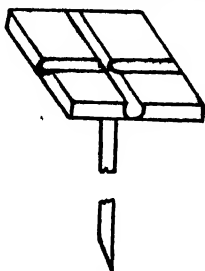*Cross-staff* or the *cross* is used to find the foot of the perpendicular from an object, to the chain line, in taking right offsets. It consists of a square piece of wood, about 6 inches square, $1\frac{1}{2}$ inches thick, having two grooves, about half an inch deep, at right angles to each other. It is mounted horizontally on a 5 feet stand, sufficiently high for easy use and which can be driven, when desired, into the earth. It is held over the stretched chain line approximately at the point from which the offset is to be taken, with one groove directed along the line of the chain. The instrument, so held, is moved along the chain until the object from which the offset is to be taken, is seen through the other groove. It is then fixed on the ground and the offset measured. The work with this instrument is not, however expeditious. Other forms of cross-staffs require looking through vertical slits, properly placed, instead of looking throuh grooves.
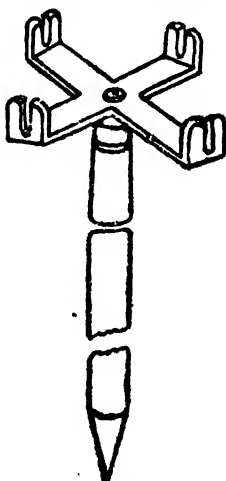
Fig. 91

Fig. 92

The *Optical Square* or the *Right angle* is a hollow
wedge-shapped box, (Fig. 93) of about 2 inches vertical
sides and 1¼ inch deep, having a handle about 3 inches
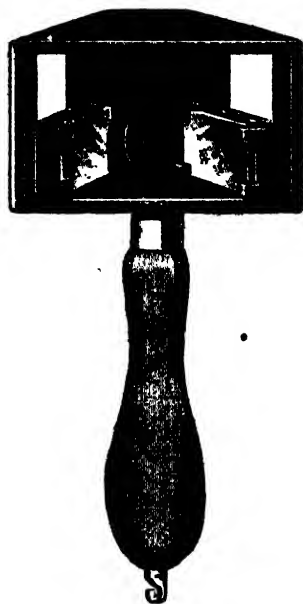

. **Fig. 93**

long fixed below it. The sides **AB, AC** (Fig. 93) are at
45 degrees to one another and has small mirrors fixed
on the inside, the other side **BC** being open. The ins-
trument is so held on the base line, by trial, that light
from any object **M** of which the offset is to be taken, after
suffering to reflections on **AB** and **AC**, is seen by
the observer at **O**, in the direction of the base line

OF, after suffering a deviation of 90 degrees. The position of a plummet, hanging from the instrument, on the base line, gives the foot of the perpendicular from the object to the base line.
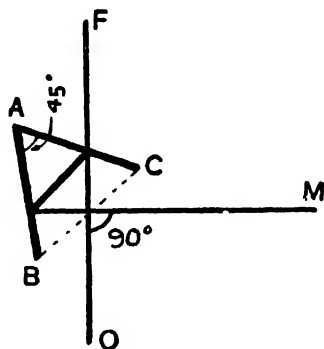
Fig. 94

The offset may also be fixed roughly in another way, with only a rope, or measuring tape, by simply holding one end, against the object **M** as centre and describing by trial a circle to touch the line ; the point of contact is the shortest distance between the object and the chain line, which is the required offset. For small lengths this is a simple and satisfactory method.
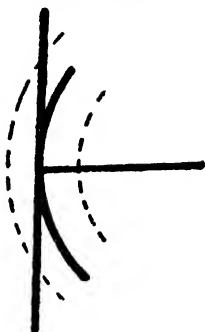
Fig. 95

(*d*) **Field-book.** Records of measurement are entered in the *field-book,* as the work proceeds. The page is divided into 3 columns by two verti-
cal lines. The records of distances
from the starting station, of offsets,
measured on the base line, are
entered in the middle and narrow
column, from the bottom, while the
lengths of offsets are entered on the
right or left columns, according as
they are situated with respect to
the base line, against the corresponding entries in the
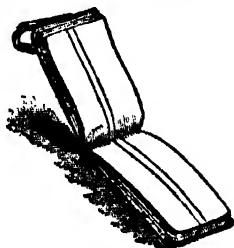middle column.

Fig. 96

This is better than the "single line" field-book, where measurements of base lines and of offsets, entered against a single line, may be confused, specially by the beginner. The only objection to the double line field-book is that lines, representing roads, drains, hedges, etc·, which are crossed by the chain line, have to be sketched in approximate position in the book and are so shown broken by the space in the middle column.

Objects at some distance from the chain line, such as rivers, trees, buildings and so on, are also shown in approximate positions. Noting of the stations chosen and of directions of chain line from the stations, should be very carefully done.

(*e*) **Map.** A map of the land can now be drawn from the records in the field-book, to any suitable scale. Areas can be found out by the ordinary rules of Mensuration.

9

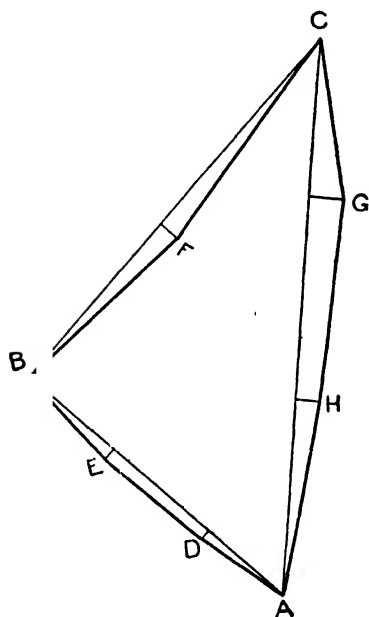**Examples 1.** Make a sketch of the field from the following notes in a field-book and work out its area :—

	Links	
	A	
0	500	
H 20	320	
G 30	140	
0	0	0
	C	Turn to right
	C	
	400	0
	180	20 F
	0	0
	B	Go N. E.
	B	
0	300	
E 12	200	
D 10	90	
	0	0
	A	Go N. W.

Fig. 97

$$\left.\begin{array}{c} \dfrac{300 + 400 + 500}{2} \\[2mm] 600 \end{array}\right\}$$

$$\left.\begin{array}{c} s - a = 300 \\ s - b = 200 \\ s - c = 100 \end{array}\right\}$$

Area of triangle **ABC**

$$= \sqrt{s(s-a)\ (s-b)\ (s-c)}$$

$$= \sqrt{600 \times 300 \times 200 \times 100}$$

$$= 60000 \text{ sq. links.}$$

Area of △**AD**. 90 = $\frac{1}{2}$(90 × 10) = 450 sq. links, to be added.

„ fig. **DE**. 200. 90 = 110 × 11 = 1210 „

„ triangle **EB**. 200 = $\frac{1}{2}$ (100 × 12) = 600 „

„ triangle **BFC** = $\frac{1}{2}$(400 × 20) = 4000 links to be subtracted.

„ „ **CG**. 140 = $\frac{1}{2}$(140 × 30) = 2100 „ to be added.

fig. **G**. 140.320. **H** = 180 × 25 = 4500 „ „

„ triangle **H**. 320. **A** = $\frac{1}{2}$(180 × 20) = 1800,, „

Total area = 60,000 + 10,660 − 4000 = 66660 sq. links.

$$= 6\cdot666 \text{ sq. inches.}$$

2. Lay off a perpendicular with the chain or tape, on the chain line from a given point **P** on it ; also lay out angles of 60°, 30° and 45°.
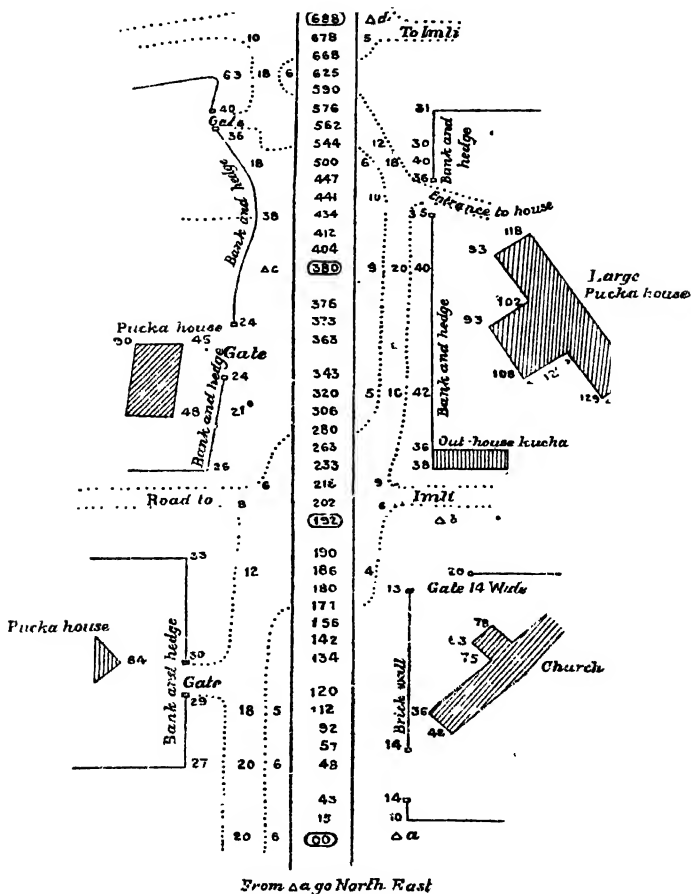
(*i*) Find a point **A**, 30 ft. from **P** on the chain line. Take 90 ft. of chain or tape and holding the ends at **A** and **P**, find the point **O**, on the stretched chain which is 50 ft. from **A** and 40 ft. from **P**.

(*ii*) Alternatively, find two points **B** and **B'** on the chain line, equidistant from **P**. Holding the ends of a certain length of chain or tape at **B** and **B'**, find the position of the middle point **O**, of this stretched chain.

OP is the perpendicular to the chain line at **P**. If in (*ii*), we make **BP** = **PB'** = 25 feet or links and take 100 ft. or links for **BOB'**, the angles at **B** and **B'** are 60° each and angles **BOP** and **B'OP** are 30° each.
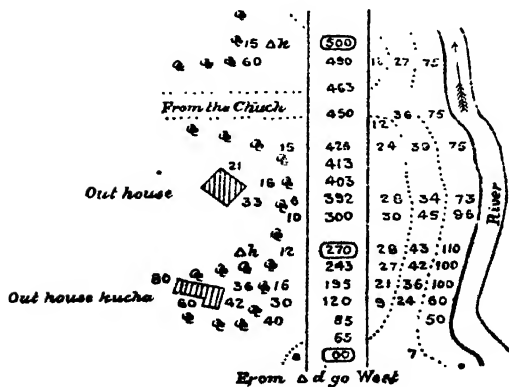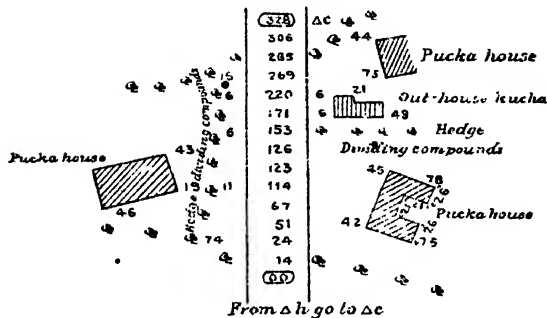
In (*i*), if **A'** is another point on the chain line at 40 ft. from **P**, angles at **O** and **A'**, of the triangle **OPA'** are each 45°.

3. The following example will indicate the way, but actual practice is necessary in learning the work.

FIELD BOOK OF A CHAIN SURVEY.

Fig. 98

Δ k ⑤⑥⑤
561
505
492

Small clump of trees ⑦⑦⑦

Entrance to compound

Large pucka house

From Δ b go to Δ k

Δ c

Pucka house

Out-house kucha

Hedge

Dwelling compounds

Pucka house

Pucka house

From Δ h go to Δ c

From the Church

Out house

Out house kucha

River

From Δ d go West

Scale 200 Feet to 1 Inch
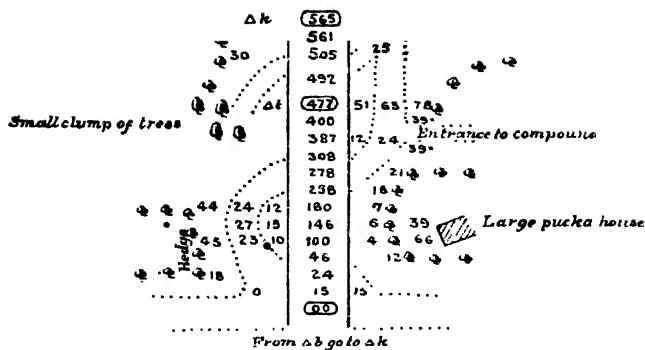
Fig. 100

EXERCISE 27.

1. Draw the plans and find the areas of the fields from the following notes ; lengths are expressed in links :—
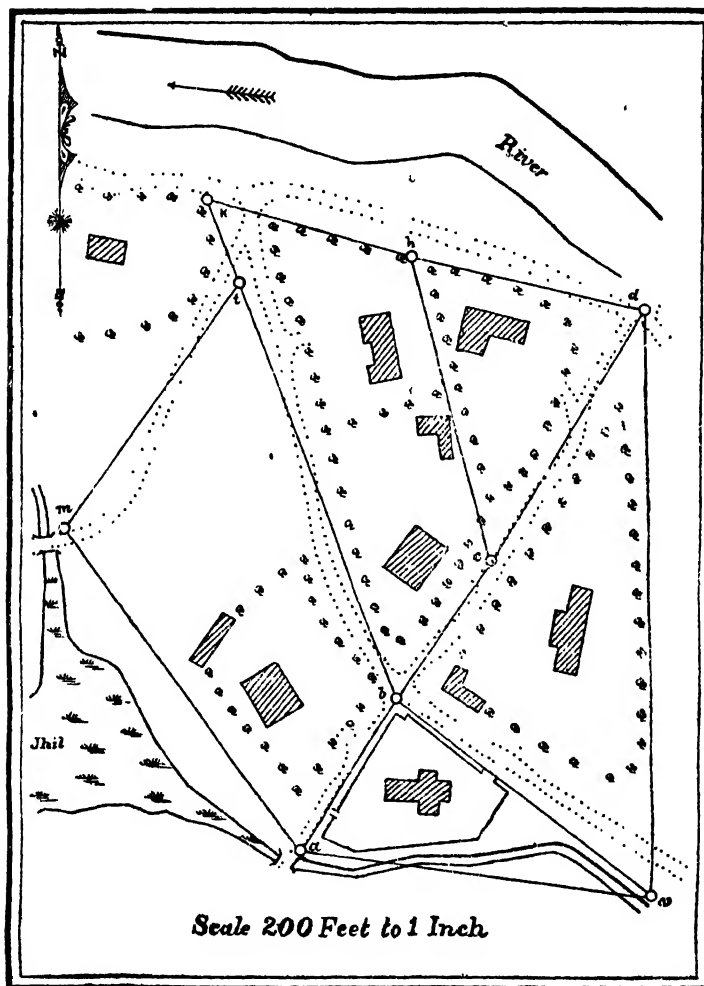
to D 100 to B 155	to **E** • 550 400 350 180 from **A**	• 110 to **C**

2

to **B** 200	to **E** 450 290 150 from **A**	80 to **D** 90 to **C**

3.

to **E** 120 to **D** 70 to **B** 150	to **F** 800 650 400 350 180 from **A**	• 110 to **C**

4.

9 4 .17	102 75 40 12 ⊙	8 12 7

6. Lay down the field **ABCDEFG**, and find its area from the following dimensions.

5.

270 220 184	1394 1112 940 614 368 160 38 0 ⊙	235 62 42

G 690	to ⊙ **D** 1560 864 618 from ⊙ **F**	20 **E**
D 690	to ⊙ **G** 1305 363 from ⊙ **C**	
B 362	1650 1230 405	
Begin at	⊙ **A**	390 **G** range East.

7.

	⊙ **A**	
	1650	0
	1300	30 **L**
D 1232	d 726	
	500	0 **K**
	260	20 **H** .
	0	0
	⊙ **G**	
turn to	the left	
	⊙ **G**	
0	1430	
F 10	820	
	600	0
	270	40 **E**
	0	0
	⊙ **D**	
turn to	the left	
	⊙ **D**	
	1540	0
	960	30 **C**
.	300	10 **B**
	0	0
from	⊙ **A**	go North East

Lay down the field from the notes and find the area of the field.

———:o:———

ANSWERS.

1. (a) 5 ;   (b) 78 ;   (c) 22 nearly ;   (d) 41 yds. 2 ft. ;
    (e)   7 ft. 1 in. ;   (f)   1 bigha 77 cu.
2   (a)   7 ;   (b)   3·61 ;   (c)   24 ;   (d)   2 ft. 4 ins.;
    (e)   3 yds ;   (f)   1 bigha 8·02 cu.
3. 16 ft.   4.   12·62 ft. nearly.
5. 637 ft. ;   245 ft.   6.   12637 ft. ;   12012 ft.
7. 8 ft.   8.   40 yds.
9. 1 ft   10. Rs. 112 nearly.
11. 6.   12. 68 ft.
13. 1 mile 1740 yds.   14. 4 ft.
15. 390 miles nearly.   16. 1 ft. 6·3 ins. nearly.
17. 21·88 ft. nearly.   18. 24·1 ft.

1. 1·41421.   2.   9 899 ins.
3. 7·07 ins. nearly.   4.   42 4 minutes nearly.
5. 4 8 inches.   6.   4·472 ins. from centre.
7. 2·4 ins   8.   0·7 ins. from centre.
9. 16·8 miles.   10. Rs. 113. 2 as. nearly.
11. 4·8 ins.   12. 7·5 ins. nearly.

1. 1·732.   2.   8 inches.
3. 4·5 inches.   4.   10 inches.
5. 300 ft.   6.   25 ft. 11·36 inches.
7. Rs. 51. 15 as. 4 p.   8.   8 feet.

9. 4·24 yds. nearly.   10. 2 sq. ft.

11. 3, 4 and 2·4 ins.   12. 15 ft.

13. 60 ft.   14. 12 ft.

15. 24 ft.   16. 22 4, 24, 25·86 yds.

17. 15 ; 99.   18. 60 ft. .

19. 17·84 ins. nearly.   20. ʿ (1) 144·2 yds ; 33·28 yds.
(2) 500 ft ; 82·368 ft.

21. 69·71 ft.   22. 20·0016.

23. (1) 37·5 ft., 36 ft.   (2) 2 ft. 1 in., 1 ft. 1·13 inches.
(3) 50 ft., 48 ft.

24. (1) 2·58 or 18·58.   (2) 52·91 ft.   (3) 22·36 ft.

EXERCISE 18.   Page 74.

1. $5\frac{1}{3}$ ft.   2. 10 ft. and 12 ft.

3. 25 ft.   4. $94\frac{6}{11}$ ft.

5. 150 ft.   6. 6 inches.

7. $4\frac{4}{5}$ inches.   8. 5·19 inches and 3 inches.

9. 13·856 inches.   10. 48·28 inches.

EXERCISE 19.   Page 80.

1. (1) $\frac{4}{7}$ inches.   (2) 1·0416 inches.   (3) 12·5 ins.
(4) 8·17 ins.   (5) 7·74 ins.   (6) 3·87 ins.
(7) 12 ins.   (8) 9 ins.   (9) 9 ins.
(10) 12 ins.   (11) $11\frac{1}{3}$ ins.   (12) 20 ins.
(13) $16\frac{2}{3}$ ins.   (14) 5·29 ins.

2. (1) 11·312 inches.   (2) 8 ins.   (3) 12 ins.
(4) 18·32 ins.   (5) 17·32 ins.   (6) 10·72 ins.

3. (1) 2·227 ins.   (2) 8 ins. nearly.   (3) 8·909 ins.
(4) 29·908 ins.   (5) 10·3409 ins.   (6) 13·681 ins
(7) 9·227 ins.   (8) 45·818 ins.

4. (1a) 2 ins.   (1b) 8·38 ins.
    (2a) 1·5 ins.   (2b) 6·28 ins.
    (3a) 1 in.   (3b) 4·19 ins.
    (4a) 2·5 ins.   (4b) 10·47 ins.
    (5a) 3 ins.   (5b) 12·57 ins.
    (6a) 4 ins.   (6b) 16·76 ins.
    (7a) 1·2 ins.   (7b) 5·028 ins.
    (8a) 3·33 ins.   (8b) 13·96 ins.

5. 8 yds. 1 ft. 1·6 ins. nearly.

6. 105 yds.   7. 14 ft.   8. 660 ft.

9. 7·002 yds.   10. 12·6 or 71·4 ins. nearly.

11. 33·8 ft.   12. 10·16 ft. ; 7·8102 ft.

13. 50·7 ft.   14. 8·74 ft.   15. 5·74 ft.

16. 10 ins.   17. 102 ft.   18. 8·484 ft

19. 14·68 ins.   20. 5082 ins.

EXERCISE 20   Page 85.

1. (a) 7 sq. yds.   (b) 11 sq yds.
    (c) 720 sq. yds.   (d) 5868 sq. yds. nearly.

2. (a) $3\frac{1}{10}$ sq. bighas.   (b) 2·06 acres.
    (c) $6\frac{24}{121}$ acres.   (d) 640 acres.

3. Rs. 384/-   4. Rs. 72. 14as. 4·8p.

5. Rs. 46933 nearly.   6. Rs. 391. 8as.

7. Rs. 360/-   8. Rs. 162/-

9. Rs. 5. 10a.   10. Rs. 1125 nearly.

11. 10833 sq. yds.   12. 9.   13. $13\frac{3}{4}$ ft.

14. 31 ft. × 21 ft. × 13 ft. height.

EXERCISE 21.   Page 89.

1. 24 sq. ft.   2. 3456 ; 60.

3. 627·39 sq ft.   4. 8 ft.

5.   346·427712 ; 20·0016.

6.   2 ft. 1 in. ; 2 sq. ft. 48 sq. ins.

7.   15 ft.   8.*   28 ft.

9.   22·7 ins.   10.   60 ft.

11.   84 sq. ft.   12.   £12·18s.

13.   317 ft. nearly.   14. $119_{2\frac{1}{1}}$ sq. yds.

15   0·537 acres.   16.   6·928.

17.   4·2 yds.   18.   £4 per acre.

19.   709 ft. nearly.   20.   £1218. 19s. $3\frac{1}{9}$d.

21.   240 yds.   22. $8\frac{1}{2}$ ins.

23.   15 ; 99.   24.   17·632 acres.

25.   216.   26.   560 sq. ft.

27.   540 sq. ins   28.   720 sq. yds.

29.   52330·3   ‘   30.   1229·8... . sq. ft.

EXERCISE 22.   Page 95.

1.   132 ft.   2.   39·242 yds.

3.   4840 yds.   4.   10 ft.

5.   12727·27 sq yds.   6.   £19. 0s. $3\frac{3}{4}$d.

7.   236 sq. yds. $82\frac{1}{7}$ sq. ins.   8.   21·46 sq. ins.

9.   36 sq. ft ; $103_{1\frac{1}{1}}$ sq. ft.   10.   15·093 ins

11.   33 in. ; 198 sq. ins.   12.   314·16 sq. ft.

13. $160_{1\frac{3}{2}}$ sq. ft.   14.   908 ; 30520 nearly.

15.   70 sq. ft.   16.   400 sq. ft.

17.   8 3   sq. ft. nearly.   18.   2141·2 sq. ins. nearly

19.   28·4 nearly.   20.   218·8 sq. ins. nearly.

21.   55 sq. ft. nearly.   22.   27·5 sq. ins. nearly.

23.   208·6 ; 100·6 nearly.   24.   0·06 sq. ins.

25.   33·5 nearly.   26.   119·318 sq. ft.

27. $117\frac{6}{7}$ sq. ft.   28.   4549·92 sq. ft.

29.	3120 sq. chains.	30.	48·989 ; 844·94.
31.	519·61 sq. ft.	32.	125·05.
33.	9 ins. ; 22½ ins.	34.	6·928 ins.
35.	17,084·8 sq. ft. nearly.	36.	293 ft.
37.	1·732 ft.	38.	200⅔⅝ ft.

EXERCISE 23.   Page 102.

1. 259·807 sq. ft.   2. 12 acres 11 poles.

3. 23636 sq. ft. nearly.   4. 293·89 sq. ins.

5. 4 : 3√3 : 6.   6. 3·25 yds. nearly.

7. 1931·3 sq. ft.   8. 1·414 ft.

9. $\frac{3}{16}$ sq. ft.   10. 4·682 ft.

11. 2·1875 acres, 500 links. 12. 1602 sq. ft.

13. 950 sq. yds.   14. 162463·9375.   19.   3168 sq. ft.

20. 924·6 sq. ft.   21. 3900 sq. ft.

22. 50√3 ; 60√3 ; 80√3. 23. 44·577...ins.

24. $\dfrac{\text{diam}}{\sqrt{3}}$ 25. 4 : 9.   26. 7·46...acres ; 269·4 yds.

27. 1 furlong = 1 in.   28. 117 ; 156 ; 195 ft.

EXERCISE 24.   Page 111.

2. 42·4 ft. nearly.

3. 9 cu. ft. ; 3182 cu. ins·   4. 314·5 cu. ft.

5. 5196·15 cu ins.   6. 17·5 ft. broad ; 13 ft. high.

7. 12288.   8. 830·2 sq. ft. nearly· 9. 974·278 cu. ft.

10. 1500 cu·ft.   11.   157,023 acres.

12. 1643 cu. ft. nearly ; 671 cu. ft. nearly.

13. 373·4 lbs. nearly.   14. 1257¼ cu. ft. ; 550 cu. ft.

15. 2710⁵⁄₇ cu. ft., Rs. 677. 11 as. nearly.

16. Rs. 5091. 7 as. nearly.   17.   2·598 cu. ft.

18. 35,982⅔ cu. yds. ; 5⅓ ft.

EXERCISE 25. Page 116.

1.	935·3 cu. ft. nearly.	2.	81·2 cu. ft. nearly.
3.	1885·6 cu. ft. nearly.	4.	166⅔ cu. ft.
5.	203·6 cu. ins. nearly.	6.	3·399 cu. ft.
7.	6363·96 cu. ft.	8.	311·7 cu. ft. nearly.
9.	37½ cu. ins.	10.	70⁵⁄₇ cu. ft.

11. 63·39 cu. ins.

12. 18·75 ft. ; 19·4 ft. ; 10·09 ft.

13.	6285 gallons nearly.	14.	339·37 cu. ins.
15.	3240 cu. ins.	16.	21 cu. ft.

EXERCISE 26. Page 118.

1.	4071·5 sq. ins. nearly.	2.	£19. 14s.
3.	1386 sq. ins.	4.	48 cu. ft. nearly
5.	£466. 4s. nearly.	6.	128·47 sq. ft.
7.	5·385 ft.	8.	18 ins.
10.	6 ft.		

EXERCISE 27. Page 135.

1.	0·798 acre.	2.	·0717 acre.
3.	1·1445 acre.	4.	1511 sq. links.
5.	3·61135 acres.	6.	16·2443 acres.
7.	10·699 acres.		

CONTENTS